Solution of ECE60L Final (Spring 2004)

Notes: 1. For each problem, 20% of points is for the “correct” final answer.
2. Messy, incoherent papers lose point!
3. Use the following information in solving or designing circuits: OpAmps are powered by ±15 V power supplies (power supplies not shown), have a unity-gain bandwidth of $10^6$ Hz, a maximum output current limit of 100 mA, and a slew rate of 1 V/μs.
NPN Si transistors have $\beta = 200$, $\beta_{\text{min}} = 100$, $r_e = 3$ kΩ, and $r_o = 100$ kΩ.
NMOS transistors have $K = 0.25$ mA/V$^2$ and $V_t = 1$ V.
In circuit design, use 5% tolarence commercial resistor and capacitor values of 1, 1.1, 1.2, 1.3, 1.5, 1.6, 1.8, 2, 2.2, 2.4, 2.7, 3., 3.3, 3.6, 3.9, 4.3, 4.7, 5.1, 5.6, 6.2, 6.8, 7.5, 8.2, 9.1 ($\times 10^n$ where $n$ is an integer). You can also use 5 mH inductors.

Problem 1. Design a passive low-pass filter with a cut-off frequency of 10 kHz. The load for this filter, $R_L \geq 10$ kΩ. (5 pts)

Prototype of the circuit is shown with

$$\omega_c = 2\pi f_c = \frac{1}{RC} = 2\pi \times 10 \times 10^3$$

$$Z_o|_{\text{max}} = R \ll 10 \text{ kΩ}$$

The second equation gives: $R \leq 0.1 \times 10 \text{ kΩ} = 1 \text{ kΩ}$. Choosing $R = 1kΩ$ (this keeps capacitor as small as possible):

$$\frac{1}{C} = 2\pi \times 10 \times 10^3 \times 10^3 \quad \rightarrow \quad C = 16 \times 10^{-9} \text{ F} = 16 \text{ nF}$$

Reasonable commercial values are $R = 1 \text{ kΩ}$ and $C = 16 \text{ nF}$. 
Problem 2. The input and output signals to TWO OpAmp circuits are given below \((T = 0.1 \text{ ms})\). Design the TWO circuits that generate these output signals (Assume OpAmps are ideal). (10 pts)

Part A: Both input and output are square waves. Output signal is inverted \((180^\circ \text{ phase shift})\) and has an amplitude that is 5 times greater than input. Therefore, the OpAmp circuit should be an inverting amplifier. The prototype of the circuit is shown with the gain of \(v_\text{o}/v_\text{i} = -R_2/R_1\). Substituting for \(v_\text{o}\) and \(v_\text{i}\), we get: \(R_2/R_1 = 5\). Choosing \(R_1 = 10 \text{ k}\Omega\), we get \(R_2 = 50 \text{ k}\Omega\) (commercial values of 10 and 51 k\Omega).

Part B: The input signal is a square wave and the output is a triangular wave. Therefore, the circuit should include an integrator. The prototype of the circuit is shown with

\[
\frac{v_\text{a}(t) - v_\text{a}(0)}{v_\text{i}(t_0)} = -\frac{1}{R_1C_2} \int_0^t v_\text{i}(t')dt'
\]

Considering the time period between 0 and \(T/2\), we get:

\[
v_\text{a}(t = T/2) - v_\text{a}(0) = -\frac{1}{R_1C_2} \int_0^{T/2} v_\text{i}(t')dt' + 1 - (-1) = \frac{1}{R_1C_2} \int_0^{T/2} (-1)dt' = \frac{1}{R_1C_2} \times \frac{T}{2}
\]

\[
R_1C_2 = \frac{10^{-4}}{4} = 2.5 \times 10^{-5}
\]

Choosing \(R_1 = 1 \text{ k}\Omega\), we get \(C_2 = 2.5 \times 10^{-8} \text{ F} = 25 \text{ nF}\).

We have to choose \(R_2\) such that \(\tau = R_2C_2\) is about 100 times the period of lowest frequency on interest. As only one frequency is given, we use that value:

\[
\tau = R_2C_2 = 100T = 10^{-2} \quad \rightarrow \quad R_2 = \frac{10^{-2}}{2.5 \times 10^{-8}} = 400 \text{ k}\Omega
\]

Commercial values: \(R_1 = 1 \text{ k}\Omega\), \(R_2 = 390 \text{ k}\Omega\), \(C_2 = 24 \text{ nF}\).
**Problem 3.** Consider the BJT circuit shown. Find the bias point parameters ($I_C$, $I_B$, and $V_{CE}$) for both transistors (For this problem only assume $\beta_{Q1} = 100$ and $\beta_{Q2} = 50$). (10 pts)

To solve the circuit, we first replace the biasing resistors of $Q_1$ with their Thevenin equivalent:

$$R_B = 82 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 8.9 \text{ k}\Omega$$

$$V_{BB} = \frac{10 \times 10^3}{10 \times 10^3 + 82 \times 10^3} \times 24 = 2.61 \text{ V}$$

Writing a KVL containing $V_{BB}$ and base-emitter junction of $Q_1$ and $Q_2$, we get:

$$V_{BB} = R_B I_{B1} + v_{BE1} + v_{BE2} + 100 I_{E2}$$

Assume $Q_1$ is in cut-off. Then, $I_{B1} = I_{E1} = 0$. This leads to $I_{B2} = 0$ and $Q_2$ being in cut-off. For the same reason, if $Q_2$ is in cut-off, $Q_1$ will be in cut-off. If both BJTs are in cut-off, the above KVL leads to $v_{BE1} + v_{BE2} = V_{BB} = 2.6 \text{ V}$. Obviously we cannot have $V_{BE} < v_\gamma = 0.7 \text{ V}$ for both BJTs and $v_{BE1} + v_{BE2} = 2.6 \text{ V}$. Therefore, both BJTs should be ON (active-linear or saturation) and $v_{BE1} = v_{BE2} = 0.7 \text{ V}$. Assume both BJTs are in active-linear. Then:

$$I_{E2} = \beta_2 I_{B2} = \beta_2 I_{E1} = \beta_1 \beta_2 I_{B1}$$

$$V_{BB} = 2.6 = 8.9 \times 10^3 I_{B1} + 0.7 + 0.7 + 100 \times 100 \times 50 I_{B1}$$

$$I_{B1} = 2.4 \mu\text{A} \quad \rightarrow \quad I_{C1} \approx I_{E1} = \beta_1 I_{B1} = 240 \mu\text{A}$$

$$I_{B2} = I_{E1} = 240 \mu\text{A} \quad \rightarrow \quad I_{C2} \approx I_{E2} = \beta_2 I_{B2} = 12 \text{ mA}$$

$V_{CE1}$ can be found from a KVL through CE terminals of $Q_1$:

$$24 = V_{CE1} + V_{BE2} + 100 I_{E2} \quad \rightarrow \quad V_{CE1} = 24 - 0.7 - 100 \times 12 \times 10^{-3} = 22.1 \text{ V}$$

As $V_{CE1} > v_\gamma$, $Q_1$ is indeed in active linear region.

$V_{CE2}$ can be found from a KVL through CE terminals of $Q_2$:

$$24 = 1,000 I_{C2} + V_{CE2} + 100 I_{E2} \quad \rightarrow \quad V_{CE2} = 24 - 1,000 \times 12 \times 10^{-3} = 10.8 \text{ V}$$

As $V_{CE2} > v_\gamma$, $Q_2$ is also in active linear region.

Therefore, the bias parameters are: $I_{B1} = 2.4 \mu\text{A}$, $I_{C1} \approx I_{E1} = I_{B2} = 240 \mu\text{A}$, $I_{C2} \approx I_{E2} = 12 \text{ mA}$, $V_{CE1} = 22.1 \text{ V}$ and $V_{CE2} = 10.8 \text{ V}$.
Problem 4. Find \( v_o \) for a) \( v_1 = v_2 = 0 \) and b) \( v_1 = V_{DD} \) and \( v_2 = 0 \). (10 pts)

1) GS-KVLs: \( v_{GS1} = v_1, v_{GS2} = v_2, \)
\( v_{GS3} = v_2 - v_{DS4} - V_{DD}, \) and \( v_{GS4} = v_1 - V_{DD}, \)
2) by KCL \( i_{D1} + i_{D2} = i_{D3} = i_{D4} \)
3) by KVL \( v_o = v_{DS1} = v_{DS2} = V_{DD} + v_{DS3} + v_{DS4}. \)

Part A: \( v_1 = v_2 = 0 \)
We first find \( v_{GS} \) and state of all transistors by using GS-KVLs in above.

\[
\begin{align*}
v_{GS1} &= v_1 = 0 < \bar{V}_t & \rightarrow & M1 \text{ is OFF} & \rightarrow & i_{D1} = 0 \\
v_{GS2} &= v_2 = 0 < \bar{V}_t & \rightarrow & M2 \text{ is OFF} & \rightarrow & i_{D2} = 0 \\
v_{GS3} &= v_2 - v_{DS4} - V_{DD} = -v_{DS4} - V_{DD} & \rightarrow & M3 \text{ is ON} & \rightarrow & i_{D3} = 0 \rightarrow v_{DS3} = 0 \\
v_{GS4} &= v_1 - V_{DD} = -V_{DD} < -\bar{V}_t & \rightarrow & M4 \text{ is ON} & \rightarrow & i_{D4} = 0 \rightarrow v_{DS4} = 0 \\
\end{align*}
\]

Finally, from KVLs in no. 3. above, we have \( v_o = V_{DD} + v_{DS3} + v_{DS4} = V_{DD}. \)

Part B: \( v_1 = V_{DD} \) and \( v_2 = 0 \)
We first find \( v_{GS} \) and state of all transistors by using KVLs in no. 1 above.

\[
\begin{align*}
v_{GS1} &= v_1 = V_{DD} > \bar{V}_t & \rightarrow & M1 \text{ is ON} \\
v_{GS2} &= v_2 = 0 < \bar{V}_t & \rightarrow & M2 \text{ is OFF} & \rightarrow & i_{D2} = 0 \\
v_{GS3} &= v_2 - v_{DS4} - V_{DD} = -v_{DS4} - V_{DD} & \rightarrow & M3 \text{ is ON} & \rightarrow & i_{D3} = 0 \rightarrow v_{DS3} = 0 \\
v_{GS4} &= v_1 - V_{DD} = 0 > -\bar{V}_t & \rightarrow & M4 \text{ is OFF} & \rightarrow & i_{D4} = 0 \\
\end{align*}
\]

Substituting for \( i_{D2} = i_{D4} = 0 \) in the KCL above, we get: \( i_{D1} = i_{D3} = 0 \). Since \( i_{D1} = 0 \) and M1 is ON, M1 should be in ohmic and \( v_{DS1} = 0 \). Then, from KVL in no. 3 above, we have \( v_o = 0. \)
**Problem 5.** Design a BJT amplifier with a gain of 3 and a cut-off frequency of 20 Hz. Set the operating point (Q-point) parameters to be $V_{CE} = 12$ V and $I_c = 4$ mA. The circuit is to be biased with a 20 V power supply. (10pts)

The prototype of this circuit is a common emitter amplifier with an emitter resistance.

**DC bias:** The power supply voltage is given: $V_{CC} = 20$ V.

$$V_{CC} = R_CI_C + V_{CE} + R_EI_E$$

$$20 - 12 = 4 \times 10^{-3}(R_C + R_E) \quad \rightarrow \quad R_C + R_E = 2.0 \, \text{k}\Omega$$

$$A_v = \frac{R_C}{R_E} = 3$$

$$4R_E + R_E = 2.0 \, \text{k}\Omega \quad \rightarrow \quad R_E = 500 \, \Omega, \ R_C = 1.5 \, \text{k}\Omega$$

Check for good biasing $V_E > 1$ V: $V_E = R_EI_E = 500 \times 4 \times 10^{-3} = 2 > 1$, it is OK.

**$R_B$ and $V_{BB}$:**

$$R_B \ll (\beta + 1)R_E \rightarrow R_B \approx 0.1\beta_{min}R_E = 0.1 \times 100 \times 500 = 5.0 \, \text{k}\Omega$$

**KVL:**

$$V_{BB} = R_BI_B + V_{BE} + R_EI_E$$

$$V_{BB} = 5.0 \times 10^3 \times \frac{4 \times 10^{-3}}{200} + 0.7 + 500 \times 4 \times 10^{-3} = 2.8 \, \text{V}$$

**$R_1$ and $R_2$:**

$$R_B = R_1 || R_2 = \frac{R_1R_2}{R_1 + R_2} = 5. \, \text{k}\Omega$$

$$\frac{V_{BB}}{V_{CC}} = \frac{R_2}{R_1 + R_2} = \frac{2.8}{20} = 0.14$$

leading to $R_1 = 35.7 \, \text{k}\Omega$ and $R_2 = 5.8 \, \text{k}\Omega$.

The value of coupling capacitor is found from:

$$R_i \approx R_B = 5. \, \text{k}\Omega$$

$$2\pi f_i = \frac{1}{R_iC_c} \quad \rightarrow \quad C_c = \frac{1}{2\pi f_iR_i} = \frac{1}{2\pi 20 \times 5,000} = 1.6 \, \mu\text{F}$$

Design values are: $R_1 = 36 \, \text{k}\Omega$, $R_2 = 5.6 \, \text{k}\Omega$, $R_E = 510 \, \Omega$, $R_C = 1.5 \, \text{k}\Omega$, and $C_c = 1.6 \, \mu\text{F}$.
Problem 6. Find $v_o/v_i$. (10 pts)

Both OpAmps have negative feedback, so $v_i = v_{p1} = v_{n1}$ and $v_{p2} = v_{n2}$. Then, by node-voltage method:

\[
\frac{v_{n1}}{R_1} - \frac{v_{n1} - v_{n2}}{R_2} = 0
\]
\[
\frac{v_{n2} - v_{n1}}{R_2} + \frac{v_{n2} - v_o}{R_2} = 0 \quad \Rightarrow \quad v_o = 2v_{n2} - v_{n1}
\]

where we have multiplied the 2nd equation by $R_2$. Substitute for $v_{n1} = v_i$ and finding $v_{n2}$ from the first equation:

\[
\frac{v_{n2}}{R_2} = \frac{v_{n1}}{R_1} + \frac{v_{n1}}{R_2} = v_i \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad \Rightarrow \quad v_{n2} = v_i \left( \frac{R_2}{R_1} + 1 \right)
\]

Substituting for $v_{n2}$ in the second equation:

\[
v_o = 2v_i \left( \frac{R_2}{R_1} + 1 \right) - v_i \quad \Rightarrow \quad \frac{v_o}{v_i} = \frac{2R_2}{R_1} + 1
\]

**Alternative Method:** From the figure, since $I_{n1} = I_{n2} = 0$, by KCL current $i$ will flow through $R_1$ and both $R_2$s. Thus, we have

\[
v_o = (2R_2 + R_1)i
\]
\[
v_i = v_{n1} = R_1 i
\]

Dividing both equations we get:

\[
\frac{v_o}{v_i} = \frac{2R_2 + R_1}{R_1} = \frac{2R_2}{R_1} + 1
\]
Problem 7. (A) Find the Q-point parameters of both BJTs in the circuit shown (Hint: Assume $I_{B2} \ll I_{E1}$ to solve and then check the validity of your assumption). (15 pts) (B) Find overall gain, input resistance, and the lower cut-off frequency of the circuit.

Part A: Replacing the biasing resistors $R_1$ and $R_2$ with its Thevenin equivalent, we get:

$$R_B = R_1 \parallel R_2 = \frac{20 \times 10^3 \times 82 \times 10^3}{20 \times 10^3 + 82 \times 10^3} = 16 \text{ k}\Omega$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{20 \times 10^3}{20 \times 10^3 + 82 \times 10^3} \times 20 = 3.9 \text{ V}$$

By KCL: $I_{E1} = I_{B2} + I_1$. Assuming, $I_{B2} \ll I_{E1}$, we will have $I_{E1} \approx I_1$. KVL in BE loop of $Q_1$ results in:

$$V_{BB} = R_B I_{B1} + V_{BE1} + 10^3 I_1 = R_B \frac{I_{E1}}{\beta + 1} + V_{BE1} + 10^3 I_{E1}$$

$$3.9 = 0.7 + I_{E1} \left( \frac{16 \times 10^3}{200 + 1} + 10^3 \right)$$

$$I_1 \approx I_{E1} \approx I_{C1} = 3.0 \text{ mA} \quad \rightarrow \quad I_{B1} = \frac{I_{C1}}{\beta} = 15 \mu\text{A}$$

Where we have assumed that $Q_1$ is in active region. KVL in CE loop of $Q_1$ results in:

$$20 = V_{CE1} + 10^3 I_1 = V_{CE1} + 3.0 \times 10^{-3} \times 10^3 = V_{CE1} + 3.0 \quad \rightarrow \quad V_{CE1} = 17.0 \text{ V}$$

Since $V_{CE1} > v_r$, our assumption of $Q_1$ in active is correct. So the bias point values are $Q_1$: $I_{E1} \approx I_{C1} = 3.0 \text{ mA}$, $I_{B1} = 15 \mu\text{A}$, and $V_{CE1} = 17.0 \text{ V}$.

KVL in BE loop of $Q_2$ results in (assuming $Q_2$ is in active region):

$$-10^3 I_1 + V_{BE2} + 500 I_{E2} = 0 \quad \rightarrow \quad -10^3 \times 3.0 \times 10^{-3} + 0.7 + 500 I_{E2} = 0$$

$$I_{C2} \approx I_{E2} = 4.6 \text{ mA} \quad \rightarrow \quad I_{B2} = \frac{I_{C2}}{\beta} = 23 \mu\text{A}$$

We note that $I_{B2} = 23 \mu\text{A} \ll I_{E1} = 3.0 \text{ mA}$, so our assumption was valid.

KVL in CE loop of $Q_2$ results in:

$$20 = 2 \times 10^3 I_{C2} + V_{CE2} + 500 I_{E2} = 2 \times 10^3 \times 4.6 \times 10^{-3} + V_{CE2} + 500 \times 4.6 \times 10^{-3}$$

$$20 = 9.2 + V_{CE2} + 2.3 \quad \rightarrow \quad V_{CE2} = 8.5 \text{ V}$$
Since $V_{CE2} > v_\gamma$, our assumption of $Q_2$ in active is correct. So the bias point values are $Q_2$: $I_{E2} \approx I_{C2} = 4.6$ mA, $I_{B1} = 23$ $\mu$A, and $V_{CE2} = 8.5$ V.

**Part B:**

The second stage is a common emitter amplifier. Therefore:

$$A_2 \approx \frac{R_{C2}}{R_{E2}} = -\frac{2,000}{500} = -4$$

$$R_{i2} = R_{B2} \parallel [\beta(R_{E2} + r_e)] = \beta(R_{E2} + r_e) \approx \beta R_{E2} = 100 \text{ k\Omega}$$

because there are no bias resistors and, therefore, $R_{B2} \rightarrow \infty$. Also, since there is no coupling capacitor before the second stage, the 2nd stage has no lower cut-off frequency and overall cut-off frequency is set by the 1st stage.

The first stage is an emitter follower. The input resistance of the second stage appears in parallel to $R_{E1}$, i.e., $R_E$ should be replaced with $R'_{E1} = R_{E1} \parallel R_{i2}$.

As $R_{i2} = 100 \text{ k\Omega} \gg R_{E1} = 1 \text{ k\Omega}$, $R'_{E1} \approx R_{E1}$. Then:

$$A_1 \approx 1$$

$$R_{i1} \approx R_{B1} = 16 \text{ k\Omega}$$

$$\omega_{l1} = 2\pi f_{l1} = \frac{1}{R_{i1}C_c} \rightarrow f_{l1} = 21 \text{ Hz}$$

The overall gain of the circuit is $A = A_1 \times A_2 = -4$,

The input resistance of the overall circuit is $R_i = R_{i1} = 16 \text{ k\Omega}$.

The lower cut-off frequency of the overall circuit is $f_l = f_{l1} = 21$ Hz (since the 2nd stage has no lower cut-off frequency).