Attractive 2- and 3-FP Plasma and Coil Configurations – Recent Configuration Development Results

Long-Poe Ku¹ & Paul Garabedian²

¹Princeton Plasma Physics Laboratory
²Courant Institute of Mathematical Sciences, New York University

ARIES Project Meeting, February 24-25, 2005
General Atomic, San Diego, CA
Topics of Discussion

• Development of A~2.5, two field-period configurations
  – MHH2-1104, physics characteristics and coil design
  – MHH2-K14, physics characteristics and coils

• Development of A~4.5, three field-period configurations
  – KQ26Q, physics characteristics and coil design

• Summary and Plans
MHH2
1104 and K14

Plane and perspective view of the last LCMS geometry and |B| in real space.
MHH2-1104 is a two-field period, aspect ratio 2.64 configuration whose ripple characteristic renders excellent confinement for $\alpha$ particles.
The magnetic spectrum of MHH2-1104 indicates rather low “noise” levels, particularly near s=0.5, although there is a large mirror component at the core. The overall effective ripple is very good, being 0.35% at the edge. As a result, the $\alpha$ energy loss is very small (~2% in our model calculation using ORBIT3D).

\[
B = \sum B_{mn} \cos(m\theta - [n - um]\phi)
\]

\[
\text{noise} = \sqrt{\frac{\text{magnetic energy, nonsymmetric components}}{\text{magnetic energy, symmetric components}}}
\]
The overall rotational transform at 5% $\beta$ prescribed in the configuration optimization is monotonically decreasing from 0.64 to 0.54 (thus avoiding the lowest order m=3, 4, 5 resonance). The transform due to the plasma shaping alone is also monotonically decreasing from 0.37 to 0.34.
One consequence of imposing the prescribed rotational transform (at 5% $\beta$) is to suppress, almost entirely, the $B(2,1)$ and $B(3,2)$ components in the magnetic spectrum, making QA much better for $s > 0.5$. The enhanced presence of the principal mirror component does not harm the $\alpha$ confinement, as we’ve seen in numerous occasions.
Correspondingly, the effective ripple (calculated by the NEO code for $1/\nu$ transport) for the finite $\beta$ is significantly lower in regions of $r/a>0.5$. 
Plots of magnetic field strength along several segments of the field lines also indicate that there are fewer number of secondary ripple wells, particularly in regions where $r/a<0.7$.

$|B|$ versus poloidal angle $\theta$ in radians along field lines starting @ $\varphi=0$, $\theta=0$.  

---

LPK_022405
Another consequence of the prescribed rotational transform is that the configuration has high quality flux surfaces, as shown below from a PIES calculation.

MHH2-1104 @ 5% $\beta$ with monotonically decreasing iota profile. Poincaré plot viewed in $r$-$\theta$ coordinates at $\varphi=0$.

Poincaré plots of MHH2-1104 viewed in Cartesian coordinates at three different toroidal angles.
The prescribed rotational transform profile requires a nearly flat plasma current density which means there is a need to deploy externally driven sources (hybrid approach).

Rotational transform versus normalized toroidal flux based on the current density shown on the left. \( I_p/R_B \sim 0.21 \) MA/m-T

Normalized toroidal current density versus normalized toroidal flux
An initial coil design (as of 12/07/04) for MHH2-1104 with zero pressure using “NESCOIL” approach gives reasonably smooth contours of current potential on the winding surface and a good winding surface to plasma LCMS separation.

Contours of current potential for I(pol)=1, I(tor)=0.
A proposed design for the modular coils is to have 8 coils per period with 4 types of coils. These coils have reasonably smooth winding, but there are sections where coil to coil spacing may be a little tight.

Coils of equal current viewed on the “U-V plane” of the winding surface in one field period.
Even with this relatively simple and smooth design the excursion of the coils makes them cross both the full and half-period toroidal boundaries.

Three different views of modular coils and the LCMS for the whole torus.
These coils provide a reasonable re-construction of the shape of the LCMS of the “fixed-boundary” plasma.
Comparison of the magnetic spectrum with zero pressure shows that the “free-boundary” plasma is noisier, having increased magnitudes of the non-axisymmetric components.
Correspondingly, the effective ripple (without pressure) is more than doubled in most of the plasma region. Further optimization of the coils is needed to regain the good confinement of the $\alpha$ particles.
MHH2-K14 is a configuration of the same ultra-low A family but it has a rising rotational transform profile in configuration optimization consistent with that expected with the bootstrap current and without any other driven currents.

LCMS in four toroidal angles over half period. Rotational transform as function of toroidal flux.

Expected at 5% $\beta$ with NCSX-like pressure/current profile

External transform due to plasma shaping

Assumed in configuration optimization
MHH2-K14 has reasonably good QA, but it is not as good as 1104. The B(2,1) and B(3,2) components remain to be significant in the magnetic spectrum. The loss of $\alpha$ energy is still reasonable, being $< 10\%$ ($\sim 6\%$ in one slowing down time in our model calculation).
Plots of $|B|$ along field lines show an increased amount of secondary ripples and the epsilon-effective (calculated by the NEO code) at the edge is now $\sim 0.8\%$. 

$|B|$ versus poloidal angle $\theta$ in radians along field lines starting @ $\phi=0$, $\theta=0$. 

LPK_022405
A vacuum magnetic well, \(~4\% \, @ \, s=1\), was imposed as one of the constraints in the configuration optimization.

Magnetic well depth as function of normalized toroidal flux.

Total @ 5% $\beta$, $p \propto (1-s^{1.5})^{1.5}$

From plasma shaping.
Well depth=3.8% @s=1.
But it is slightly unstable to both low- and high-n internal modes at $\beta=4\%$. 

Low-n modes $\gamma \cdot R/\nu_A \sim 0.0009$

Infinite-n ballooning modes (Cobra calculation)

$$p \propto (1-s^{1.5})^{1.5}$$
MHH2-K14 may be also unstable to the external modes for $\beta > 5\%$ according to the Terpsichore calculation, primarily due to modes of intermediate toroidal mode numbers 5 and 7.
While islands of the lowest orders, $m=3, 4, 5$, do not contribute to significant flux loss, the proximity of islands of intermediate mode numbers degrades the quality of the flux surfaces.

MHH2-K14 @ 5% $\beta$ with linear, monotonically increasing iota profile. Poincaré plot in $r$-$\theta$ coordinates at $\phi=0$. Poincaré plots in Cartesian coordinates at three different toroidal angles.
Flux loss due to islands of the lowest order may be significant if shear is weakened for non-monotonic iota profiles as shown below for the NCSX-like pressure/current profiles.

MHH2-K14 @ 5% $\beta$ with non-monotonic iota profile. Poincaré plot in $r-\theta$ coordinates at $\phi=0$.

Poincaré plots in Cartesian coordinates at two different toroidal angles.
Different coil designs have been tried for MHH2-K14. The following using “COILOPT” approach shows one design with relatively low normal field errors on the last LCMS. The design is still inadequate to recover the $\alpha$ loss characteristic of the fixed boundary plasma. The design provides a large plasma-coil spacing ($R/\Delta_{\text{min}} = 5.0$) but the minimum coil-coil spacing may not be adequate. Additionally, certain “kinkiness” may need smoothing.

$B_n$ Error = 1.1% (average),

3.4% (max)
Here we summarize the geometric properties of the 8 modular coil designs, MHH2-1104 and MHH2-K14_V.

### MHH2-1104
- No. of Coils: 8/period
- Different Types of Coils: 4
- $R/\Delta_{\text{min}}$ (coil-plasma)=5.60
- $R/\Delta_{\text{min}}$ (coil-coil)=17.9
- $I/R-B$ (max)=0.312 MA/m-T
- Coil lengths/R = 5.91, 5.63, 5.35, 5.08
- $B(\text{max})/B(0) = 3.56$ for 0.4 m by 0.4 m square conductors.

### MHH2-K14_V
- No. of Coils: 8/period
- Different Types of Coils: 4
- $R/\Delta_{\text{min}}$ (coil-plasma)=5.01
- $R/\Delta_{\text{min}}$ (coil-coil)=17.8
- $I/R-B$ (max)=0.314 MA/m-T
- Coil lengths/R=5.75, 5.29, 5.18, 5.20
- $B(\text{max})/B(0) = 2.94$ for 0.4 m by 0.4 m square conductors.
Plane and perspective views of the last LCMS geometry and |B| in real space.
KQ26Q is a 3 field-period, aspect ratio 4.5 configuration of the SNS/LPS family in which the iota profile is selected at an operating $\beta$ such that the impact of low order resonance is minimized.

LCMS in four toroidal angles over half period. Rotational transform as function of toroidal flux.
Minimizing non-axisymmetric residues and effective ripples resulted in good quasi-axisymmetry in KQ26Q. The effective ripple @s=1 is 0.7% at 4% β.

Eight major non-symmetric components in the magnetic spectrum plotted as function of normalized toroidal flux.
Plots of $|B|$ along field lines show structures of secondary ripple wells, mostly on the high field side. The effective ripple for $1/\nu$ transport (calculated by the NEO code) at the edge is $\sim 0.7\%$.

$|B|$ versus poloidal angle $\theta$ in radians along field lines starting @ $\phi=0$, $\theta=0$.
The loss of $\alpha$ energy is acceptable, ~7% in one slowing down time in our model calculation.

Particle loss as function of time.

Cumulative particle loss.

Energy loss distribution

Scatter diagram showing distribution of lost particles in energy, toroidal and poloidal angular space on LCMS.
KQ26Q has good equilibrium flux surface quality, but the remnant of the m=4 islands may be a concern in free-boundary plasma reconstruction and in coil designs.

Equilibrium calculated by PIES @4% $\beta$.

Poincaré plot in r-$\theta$ at $\phi=0$. In Cartesian

Equilibrium calculated by VMEC
KQ26Q is stable to the m=1, n=0 vertical mode according to the Terpsichore calculation (no feedback control necessary) and is slightly unstable to both low and high-n internal modes at $\beta=4\%$.

Infinite-n ballooning modes (Cobra calculation)

Low-n modes $\gamma \cdot R/v_A \sim 0.001$

Note: stability analyses were based on the pressure and current profiles given above. Profiles may be further optimized to improve MHD stabilities to both the local and global modes.
KQ26Q may be unstable to free-boundary modes for $\beta > 4\%$ according to the Terpsichore calculation primarily due to current driven forces at the $m=3, n=2$ resonance, but it could be made stable with more flux surface shaping to improve the local shear. It may also be made more stable by choosing more optimized pressure and current profiles.

These modes may be stabilized by further shaping.
Additional shaping of the plasma to improve the stability to the external kinks also improves the stability to the ballooning modes.
But effective ripples are increased by nearly a factor of 2 for \( r/a > 0.6 \) in KQ26W, resulting in enhanced loss of \( \alpha \) particles.
Loss of $\alpha$ is nearly doubled (from 7% to 12.5%) with the additional shaping of the plasma to stabilize the “Terpsichore” kink modes.

KQ26Q

Particle loss as function of time.

Cumulative particle loss.

Energy loss distribution

KQ26W
The following illustration shows a coil design with coil aspect ratio ~6. The maximum normal field error is still too high (~8%). Coil winding looks complex but coil to coil spacing is quite good.

KQ26Q_13 Coil Characteristics:
No. of Coils: 6/period
Different Types of Coils: 3
$R/\Delta_{\text{min}}$ (coil-plasma)=5.8
$R/\Delta_{\text{min}}$ (coil-coil)=10.2
$I/R-B$ (max)=0.278 MA/m-T
coil lengths/R=4.87, 4.49, 4.52
$B(\text{max})/B(0) = 2.11$ for 0.4 m by 0.4 m square conductors
The complexity of the coils is similar to that of the NCSX type configurations with the same coil-plasma spacing. The geometric properties are also comparable.
Summary and Plans

- Two additional configurations, one of the 2 field-period MHH2 family and one of the 3 field-period SNS/LPS family, have been developed to the extent that they may be included in the systems code evaluation.

- Coil designs for these configurations have been attempted but they have not evolved to the degree mature and steady enough for engineering studies. We will concentrate on improving MHH2 coils in the next phase.

- Efforts are underway to bring one of the 3 field-period, aspect ratio 6 SNS family configurations to the same level of development as MHH2 and K26Q for the systems evaluation.