
Design Constraints for Liquid-Protected Divertors

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Team**



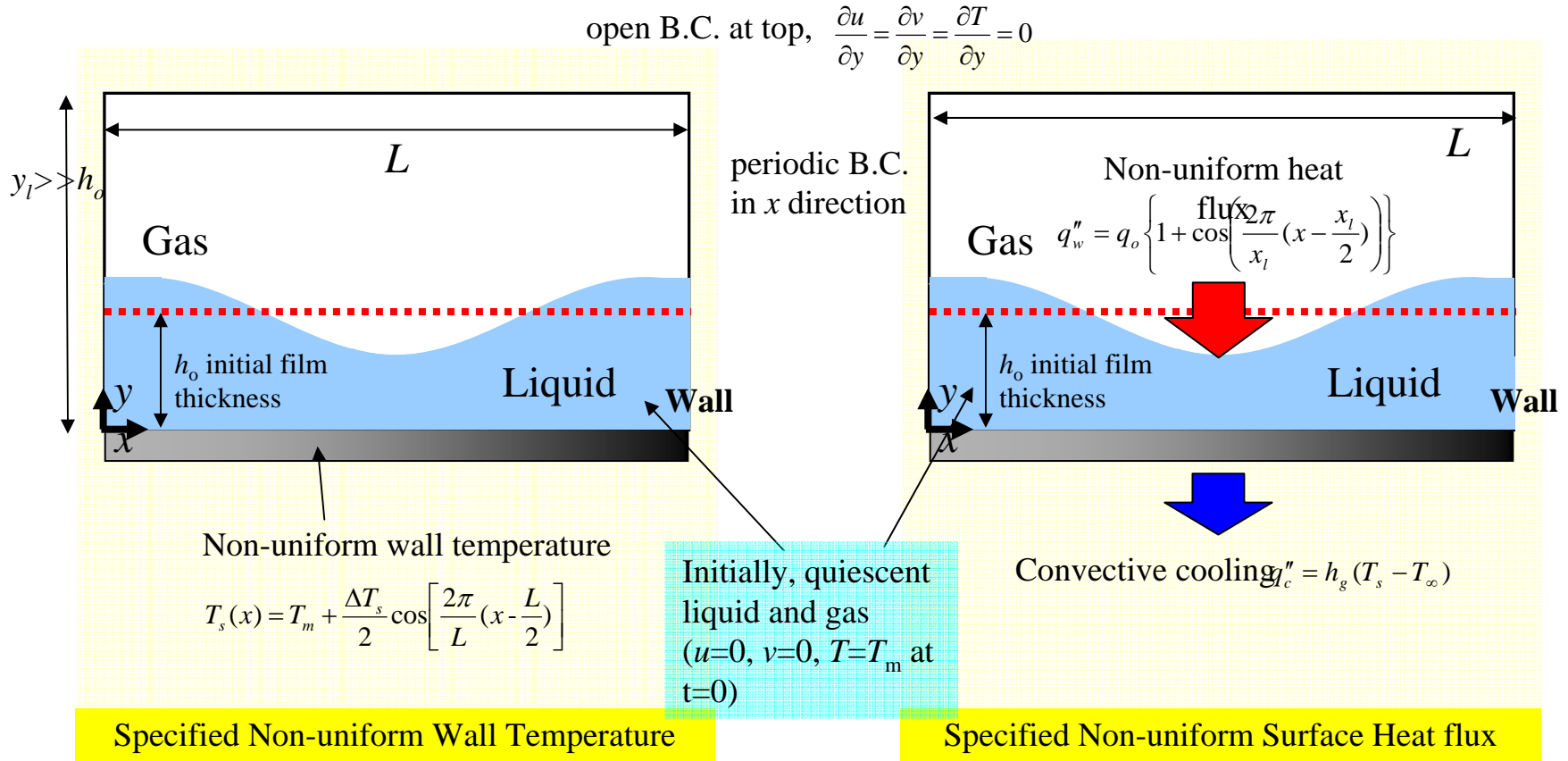
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Problem Definition

- Work on Liquid Surface Plasma Facing Components and Plasma Surface Interactions has been performed by the ALPS and APEX Programs
- Operating Temperature Windows have been established for different liquids based on allowable limits for Plasma impurities and Power Cycle efficiency requirements
- This work is aimed at establishing limits for the maximum allowable temperature gradients (i.e. heat flux gradients) to prevent film rupture due to thermocapillary effects
- Spatial Variations in the wall and Liquid Surface Temperatures are expected due to variations in the wall loading
- Thermocapillary forces created by such temperature gradients can lead to film rupture and dry spot formation in regions of elevated local temperatures
- Initial Attention focused on Plasma Facing Components protected by a “non-flowing” thin liquid film (e.g. porous wetted wall)



Problem Definition



- Two Dimensional Cartesian (x - y) Model (assume no variations in toroidal direction)
- Two Dimensional Cylindrical (r - z) Model has also been developed (local “hot spot” modeling)



Governing Equations

- Conservation of Mass $\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$

$$\sigma' = 1/We - (M/Pr)\gamma'$$

- Momentum $a^2 \rho^+ \left[\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right] = -\frac{\partial p'}{\partial x'} + a^2 \frac{\partial}{\partial x'} \left(2\mu^+ \frac{\partial u'}{\partial x'} \right) + \frac{\partial}{\partial y'} \left(\mu^+ \frac{\partial u'}{\partial y'} \right) + a^2 \frac{\partial}{\partial y'} \left(\mu^+ \frac{\partial v'}{\partial x'} \right) + \int \left(\sigma' \mathbf{k} \mathbf{n} + \frac{\partial \sigma'}{\partial s} \mathbf{t} \right) \delta ds \cdot \hat{\mathbf{i}}$

$$a^4 \rho^+ \left[\frac{\partial v'}{\partial t'} + u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} \right] = -\frac{\partial p'}{\partial y'} + \frac{\rho^+}{Fr} + a^4 \frac{\partial}{\partial x'} \left(\mu^+ \frac{\partial v'}{\partial x'} \right) + a^2 \frac{\partial}{\partial x'} \left(\mu^+ \frac{\partial u'}{\partial y'} \right) + a^2 \frac{\partial}{\partial y'} \left(2\mu^+ \frac{\partial v'}{\partial y'} \right) + a \int \left(\sigma' \mathbf{k} \mathbf{n} + \frac{\partial \sigma'}{\partial s} \mathbf{t} \right) \delta ds \cdot \hat{\mathbf{j}}$$

- Energy $a^2 \rho^+ \left[\frac{\partial c^+ T'}{\partial t'} + u' \frac{\partial c^+ T'}{\partial x'} + v' \frac{\partial c^+ T'}{\partial y'} \right] = \frac{a^2}{Pr} \frac{\partial}{\partial x'} \left(k^+ \frac{\partial T'}{\partial x'} \right) + \frac{1}{Pr} \frac{\partial}{\partial y'} \left(k^+ \frac{\partial T'}{\partial y'} \right)$

- Non-dimensional variables

$$a = \frac{h_o}{L} \quad y' = \frac{y}{h_o} \quad x' = \frac{x}{L} = \frac{ax}{h_o}$$

$$u' = \frac{u}{(\mu_L / \rho_L L)} \quad v' = \frac{v}{(a\mu_L / \rho_L L)} \quad t' = \frac{t}{(\rho_L L^2 / \mu_L)}$$

$$T' = \frac{T - T_m}{\Delta T_s} \quad V_g = \frac{\mu_L}{\rho_L h_o}$$



$$We = \frac{\rho_L V_g^2 h_o}{\sigma_o} = \frac{\mu_L^2}{\rho_L \sigma_o h_o}$$

$$Fr = \frac{V_g^2}{g h_o} = \frac{\mu_L^2}{g \rho_L^2 h_o^3}$$

$$Pr = \frac{\mu_L c_L}{k_L} \quad M = \frac{\gamma_o \Delta T_s h_o}{\mu_L \alpha_L}$$

$$q'' = -k_L \frac{\partial T}{\partial y} \quad q_o \sim k_L \frac{\Delta T}{h_o} \quad \Delta T \sim \frac{h_o q_o}{k_L}$$

$$aNu = \left(\frac{h_o}{L} \right) \left(\frac{h_g L}{k_L} \right) = \frac{h_o h_g}{k_L} \quad T' = \frac{T - T_\infty}{(h_o q_o / k_L)}$$

$$Q = 1 + \cos \left[2\pi \left(x - \frac{1}{2} \right) \right]$$

Specified Non-uniform Wall Temperature

Specified Non-uniform Surface Heat flux



Asymptotic Solution (Low Aspect Ratio Cases)

- Long wave theory with surface tension effect ($a \ll 1$). Governing Equations reduce to :

$$\frac{a^2 Fr}{We} \frac{\partial^3 h}{\partial x^3} h + \frac{\partial h}{\partial x} h + \frac{3}{2} (M/Pr) \cdot Fr \frac{\partial T_s}{\partial x} = 0 \quad \frac{\sigma_o}{\rho_L g L^2} \frac{\partial^3 h}{\partial x^3} h + \frac{\partial h}{\partial x} h + \frac{3}{2} \frac{(Q/L)}{a(\rho_L g k / \gamma_o)} \left[\frac{\partial h Q}{\partial x} + \frac{1}{B} \frac{\partial Q}{\partial x} \right] = 0$$

Specified Non-uniform Wall Temperature *

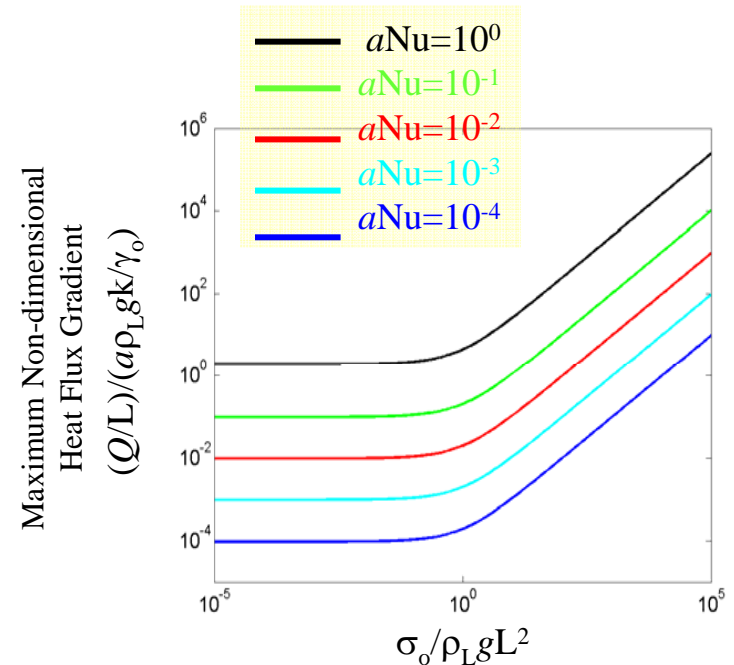
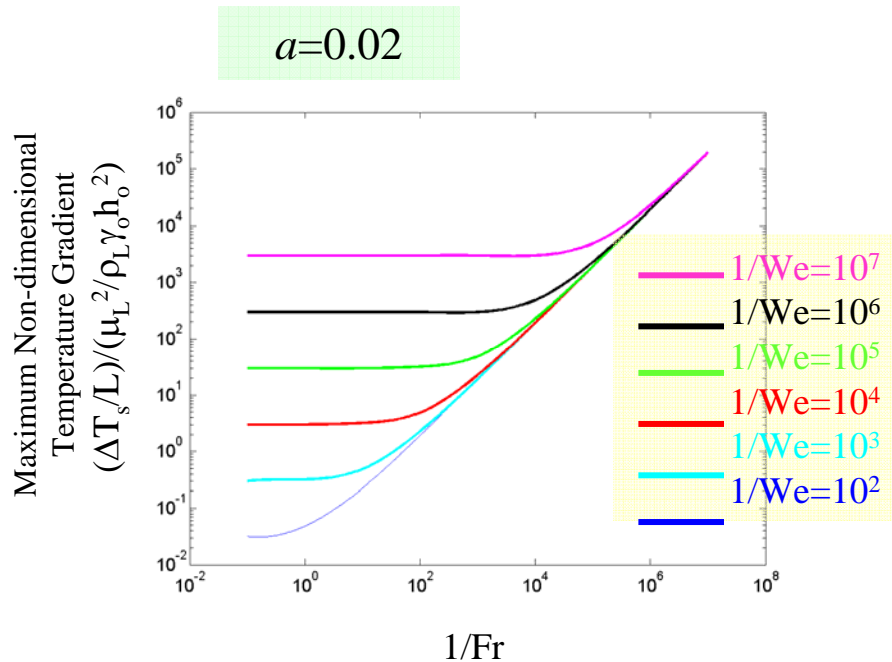
Specified Non-uniform Surface Heat flux

* [Bankoff, et al. Phys. Fluids (1990)]

- Asymptotic solution used to analyze cases for Lithium, Lithium-lead, Flibe, Tin, and Gallium with different mean temperature and film thickness
- Asymptotic solution produces conservative (i.e. low) temperature gradient limits
- Limits for “High Aspect Ratio” cases analyzed by numerically solving the full set of conservation equations using Level Contour Reconstruction Method



Asymptotic Solution (Low Aspect Ratio Cases)



Specified Non-uniform Wall Temperature

- Similar Plots have been obtained for other aspect

ratios

$$(M/Pr)_{crit} = \frac{\pi^2}{12} \frac{1}{Fr}$$

- In the limit of zero aspect ratio

Specified Non-uniform Surface Heat flux



Asymptotic Solution (Low Aspect Ratio Cases)

– Specified Non-uniform Wall Temp ($h_o=1\text{mm}$)

- Property ranges

Parameter	Lithium		Lithium-Lead		Flibe		Tin		Gallium	
	573K	773K	573K	773K	573K	773K	1073K	1473K	873K	1273K
Pr	0.042	0.026	0.031	0.013	14	2.4	0.0047	0.0035	0.0058	0.0029
1/Fr	1.2×10^4	2.2×10^5	1.9×10^5	6.3×10^5	1.2×10^3	3.8×10^4	5.3×10^5	6.7×10^5	5.7×10^5	8.2×10^5
1/We	7.8×10^5	1.3×10^6	9.4×10^5	3.0×10^6	1.3×10^4	4.0×10^5	4.2×10^6	4.8×10^6	6.9×10^6	1.0×10^7
$\frac{\mu_L^2}{\rho_L \gamma h_o^2}$ [K/m]	2.8	1.5	4.4	1.3	140	4.3	0.72	0.55	1.6	1.1

* $1/We \propto h_o$, $1/Fr \propto h_o^3$

Coolant	Mean Temperature [K]	Max. Temp. Gradient : $(\Delta T_s/L)_{\max}$ [K/cm]	
		Asymptotic Solution	Numerical Solution
Lithium	573	13	30
Lithium-Lead	673	173	570
Flibe	673	38	76
Tin	1273	80	113
Ga	1073	211	600



Asymptotic Solution (Low Aspect Ratio Cases) – Specified Non-uniform Surface Heat Flux

- $h_o=10 \text{ mm}, a=0.02$

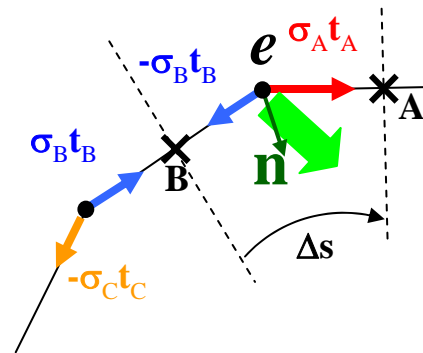
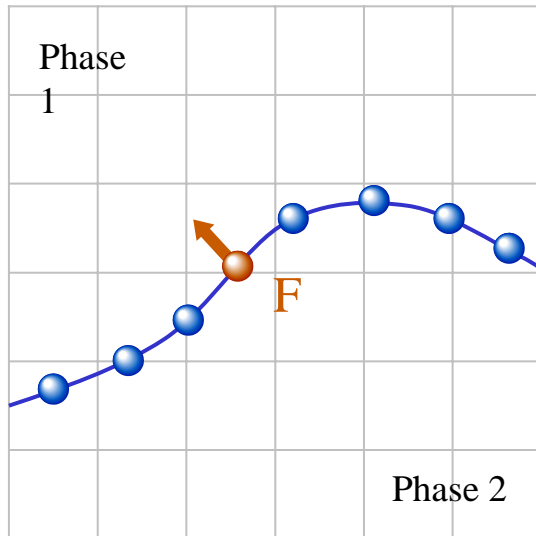
Coolant	Mean Temperature [K]	$\sigma_o/\rho_L g L^2$	Max. Heat Flux. Gradient : $(Q''/L)_{\max}$ [(MW/m ²)/cm]		
			$aNu=1.0$	$aNu=0.1$	$aNu=0.01$
Lithium	573	2.54×10^{-4}	0.61	3.1×10^{-2}	3.0×10^{-3}
Lithium-Lead	673	2.04×10^{-5}	4.9	0.24	2.4×10^{-2}
Flibe	673	4.35×10^{-5}	6.4×10^{-2}	3.2×10^{-3}	3.2×10^{-4}
Tin	1273	3.04×10^{-5}	6.8	0.34	3.3×10^{-2}
Ga	1073	4.81×10^{-5}	19	0.97	9.5×10^{-2}

- $h_o=1 \text{ mm}, aNu=1.0$

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient : $(Q''/L)_{\max}$ [(MW/m ²)/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	1.9×10^0	6.3×10^{-1}	3.1×10^{-1}	1.5×10^{-1}	6.1×10^{-2}
Lithium-Lead	673	1.2×10^1	4.9×10^0	2.4×10^0	1.2×10^0	4.9×10^{-1}
Flibe	673	1.7×10^{-1}	6.5×10^{-2}	3.2×10^{-2}	1.6×10^{-2}	6.4×10^{-3}
Tin	1273	1.7×10^1	6.8×10^0	3.4×10^0	1.7×10^0	6.8×10^{-1}
Ga	1073	5.0×10^1	1.9×10^1	9.7×10^0	4.8×10^0	1.9×10^0



Numerical Solution (High Aspect Ratio Cases)



- A single field formulation
- Constant but unequal material properties
- Surface tension included as local delta function sources

- Variable surface tension : $\sigma = \sigma_o + \gamma_o (T - T_m)$
- Force on a line element

$$\delta F_e = \int_{\Delta s} \left[\sigma \kappa \mathbf{n} + \frac{\partial \sigma}{\partial s} \mathbf{t} \right] ds = \int_B^A \frac{\partial (\sigma \mathbf{t})}{\partial s} ds = (\sigma_A \mathbf{t}_A - \sigma_B \mathbf{t}_B)$$

$$\therefore \sigma \kappa \mathbf{n} + \frac{\partial \sigma}{\partial s} \mathbf{t} = \sigma \frac{\partial \mathbf{t}}{\partial s} + \frac{\partial \sigma}{\partial s} \mathbf{t} = \frac{\partial (\sigma \mathbf{t})}{\partial s}$$

normal
surface
tension force

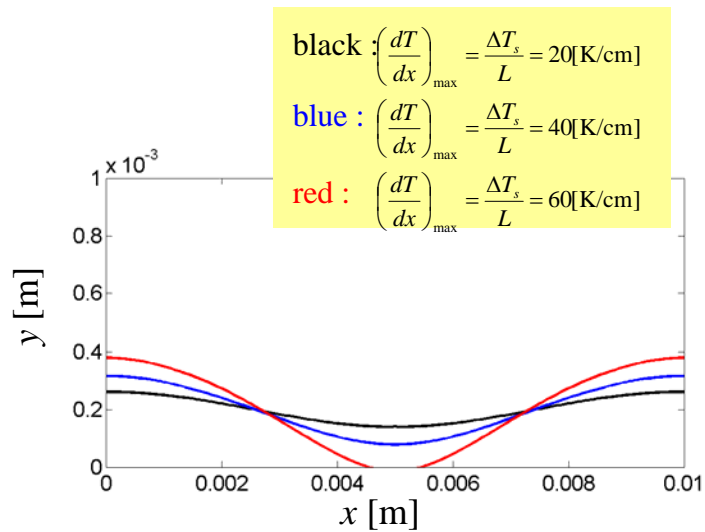
thermocapillary
force

- Evolution of the free surface is modeled using the Level Contour Reconstruction Method
- Two Grid Structures
 - Volume - entire computational domain (both phases) discretized by a standard, uniform, stationary, finite difference grid.
 - Phase Interface - discretized by Lagrangian points or elements whose motions are explicitly tracked.

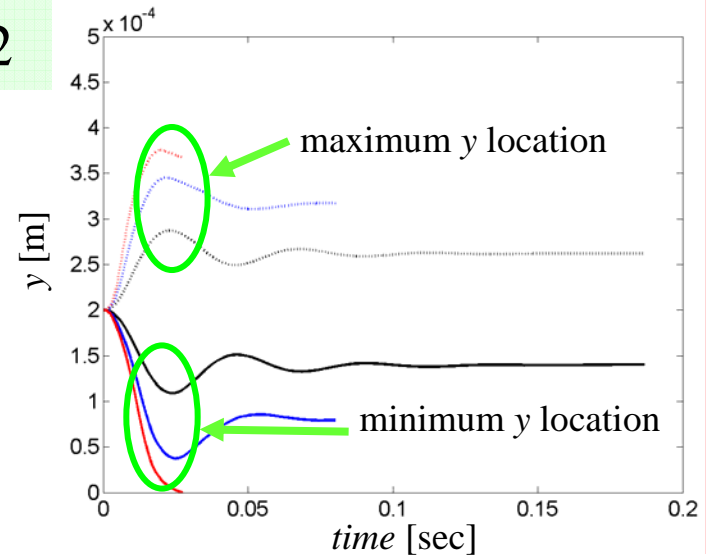


Numerical Solution (High Aspect Ratio Cases)

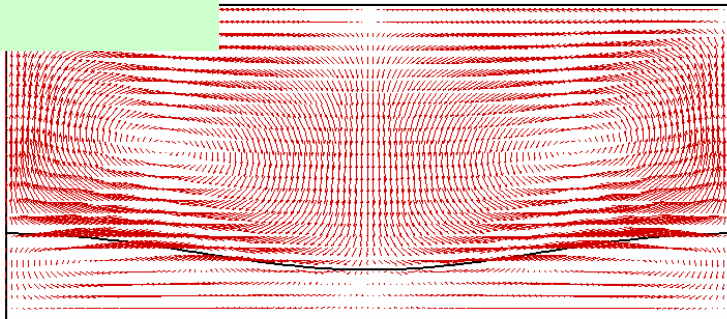
- Specified Non-uniform Wall Temperature case using Lithium



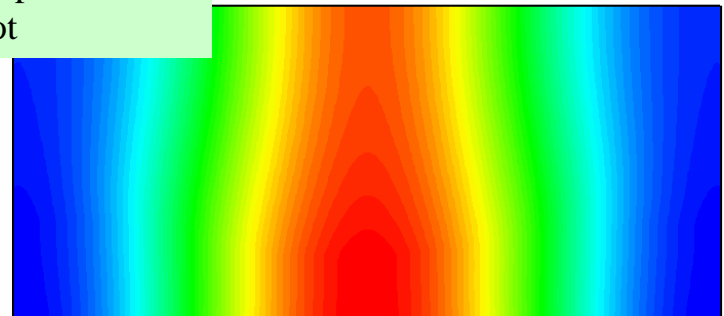
$a=0.02$



velocity vector plot



temperature plot



- Two-dimensional simulation with $1[\text{cm}] \times 0.1[\text{cm}]$ box size, 250×50 resolution, and $h_o=0.2$ mm



Numerical Solution (High Aspect Ratio Cases)

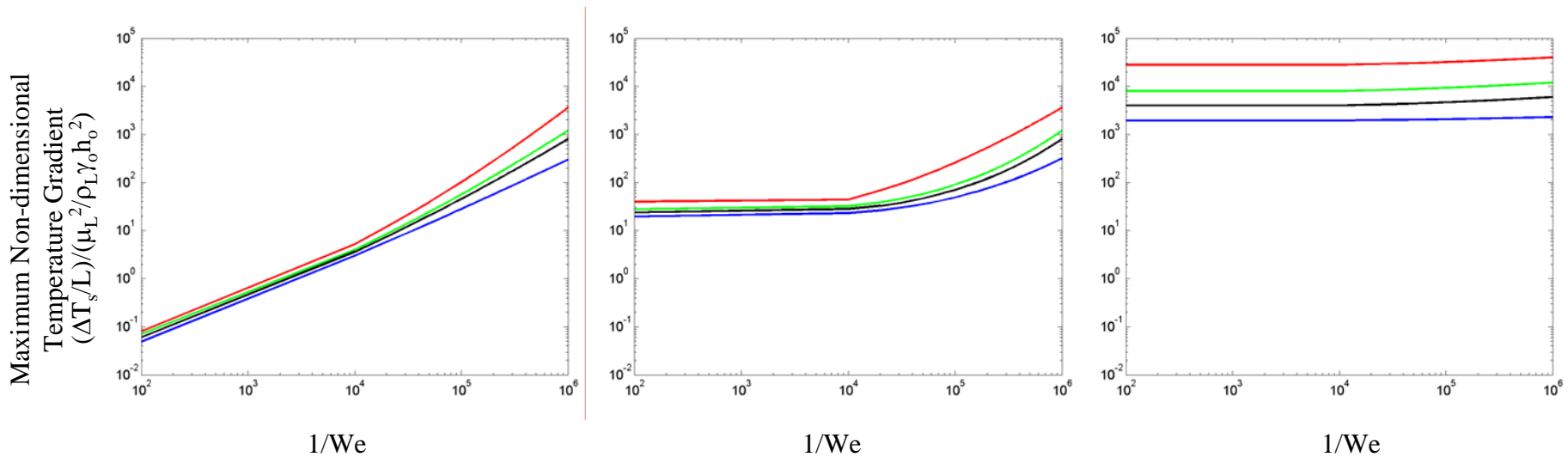
- Specified Non-uniform Wall Temperature case

$a=0.02$

$1/\text{Fr}=10^1$

$1/\text{Fr}=10^3$

$1/\text{Fr}=10^5$



— Pr=40
 — Pr=0.4
 — Pr=0.004
 — Asymptotic Solution



Conclusions

- Limiting values for the temperature gradients (or heat flux gradients) to prevent film rupture can be determined
- Generalized charts have been developed to determine the temperature (or heat flux) gradient limits for different fluids, operating temperatures (i.e. properties), and film thickness values
- For thin liquid films, limits may be more restrictive than surface temperature limits based on Plasma impurities (evaporation) constraint
- Experimental Validation of Theoretical Model has been initiated
- Preliminary results for Axisymmetric geometry (hot spot model) produce more restrictive limits for temperature gradients

