Attractive 2- and 3-FP Plasma and Coil Configurations
—
A Review of Progress and Status

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The Story Line

• Broadened the search of configuration space to find good reactor designs.
  – Improved NCSX as a reactor
  – Developed new QA configurations and corresponding coils

• Instead of focusing on a particular configuration, developed various attractive configurations to the extent that the design can be used in the systems/engineering study to understand the respective strengths and shortcomings.
  – demonstrated the richness of 3-D QA magnetic topology
  – showed the flexibility in configuration optimization

• Reactors based on these configurations are compact and competitive with other confinement concepts.
QAS power plants maybe designed with major radii <9 m (J. Lyon).
Approach and Rationale

NCSX scale-up

Coils
1) Increase plasma-coil separation
2) Simpler/"better" coils

Physics
1) Confinement of $\alpha$ particle
2) Integrity of equilibrium flux surfaces

New classes of QA configurations

MHH2
1) Develop very low aspect ratio geometry
2) Detailed coil design optimization

SNS
1) Nearly flat rotational transforms
2) Excellent flux surface quality

High leverage in reactor sizing.

Remote maintenance required in reactors.

Critical to 1st wall heat load and divertor.

Reduce consideration on MHD stability in light of W7AS and LHD results.

How compact a compact stellarator reactor can be?

How good and robust the flux surfaces one can “design”?
Configurations have been developed only at the chosen reference state; many issues have yet to be examined.

- Optimization only for the reference operating point. Startup and control generally ignored.

- Beta limits not studied. Optimization with respect to pressure/current profiles generally untouched.

- Neo-classical transport not calculated (except some MHH2). Assumed the bulk transport loss will be the anomalous (we do evaluate the $\epsilon$-effective and limit it to $< a$ few percent)

- Most configurations are of “reference” geometry; size and collisionality scaling only briefly examined.

- Configuration robustness and sensitivity to mode truncation and numerical calculations not investigated.
A few words about code calculations

- **Equilibrium**
  - VMEC, PIES, NSTAB, limited resolution.
  - Assumption for bootstrap current same as NCSX.
  - Pressure/current profiles same as NCSX, except in some 2 field period configurations where $p \sim (1-s^{1.5})^{1.5}$ used.

- **Transport**
  - Neo-classical: effective ripple, NEO
  - Alpha particles: ORBIT3D; $V=1000$ m$^3$, $B_0=6.5/5.0$ T and $n_0R/T_0^{2} \sim 0.1$, peaked $(1-s^6)$ birth distribution, parabolic background density and temperature.

- **MHD stability**
  - Ballooning: COBRA, default assumptions about convergence. two field lines starting $\theta=0$, $\phi=0$ and $\frac{1}{2}$ field period.
  - Terpsichore: 91 perturbation modes, 300 Boozer modes, $N=1$ and $N=0$ families. Wall $\sim 3x$ minor radius.

- **Coils**
  - NESCOIL, COILOPT, STELOPT
  - Limited numbers of modes used in describing winding surface, coils and free-boundary equilibria.

There are numerous calculations. There are many, many resulting numbers. While these numbers are correct only under the strict conditions and assumptions of the calculations, they should be indicative of the general characteristics of the configurations.
General Considerations of Configuration Design Targets

Minimum requirements in configuration optimization for MHD stable QA plasmas at high $\beta$ are not well known at present. The following are “acceptance criteria” generally considered:

- **Maximum residues of non-axisymmetry in magnetic spectrum.**
  - neo-classical transport \(\ll\) anomalous transport
    - **overall allowable “noise” content** \(<\sim 2\%.
    - **effective ripple in** \(1/\nu\) transport, \(\varepsilon\)-eff \(<\sim 1\%
  - ripple transport and energetic particle loss
    - **\(\alpha\) energy loss** \(<\sim 10\%
    - rotational damping (?)

- **Stability beta limits based on linear, ideal MHD theories.**
  - vertical modes
    \[
    \frac{t_{\text{ext}}}{t} \geq \frac{\kappa^2 - \kappa}{\kappa^2 + 1}
    \]
  - interchange stability
    - \(V''\sim 2-4\%.
    - LHD, CHS stable while having a hill.
• Ballooning modes
  - Stable to infinite-n modes (eigenvalues calculated by COBRA code).
  - LHD exceeds infinite-n results. High-n calculation typically gives higher $\beta$ limits.

• Kink modes
  - Stable to $n=1$ and 2 modes without a conducting wall (eigenvalues calculated by Terpsichore code).
  - W7AS results showed mode (2,1) saturation and plasma remained quiescent.

• Tearing modes
  - $\Delta < \langle \beta \rangle / \Delta < 1/2$
  - Overlapping of islands due to high shears associated with the bootstrap current.
  - Flux loss due to all isolated islands < 5%.
  - Large islands associated with low order rational surfaces.
  - Shafranov shift.

Equilibrium and equilibrium beta limits

• Shafranov shift
  - $\Delta < \langle \beta \rangle / \Delta < 1/2$
  - Limit $du/ds$

The ability to achieve our goals is often compromised by the conflicting demands of various constraints. Typically, we impose different weights depending upon the characteristics of a configuration we are looking for. There is also an issue of convergence and accuracy in numerical calculations.
To establish minimum requirements for coil design optimization, we need more feedback from and iteration with systems analysis and engineering designs. Presently, we include

- **Coil design**
  - coil to coil and coil to plasma separation
    - $R/\Delta_{\text{min}}(c-c) < 12$
    - $R/\Delta_{\text{min}}(c-p) < 6$
  - radius of curvature and complexity
    - $B_{\text{max}}/B_0 \sim 2.5$ for $0.3 \text{ m} \times 0.3 \text{ m}$ conductor @R~8 m.
  - adequate space for pumping, diagnostics, plasma heating and maintenance
    - $R/\Delta_{\text{out}}(c-p) < A_p (R/\langle a \rangle)$
I. NCSX Class of Configurations

LI383

Increase coil-plasma separation

Improve flux surface integrity and confinement of $\alpha$ particles

KQ26Q

Improve confinement of $\alpha$ particles while maintaining similar MHD stability/equilibrium surface characteristics

N3ARE
NCSX Scale-Up; Coil Improvement

\[
\begin{array}{cccc}
R/\Delta_{\text{min}}(c-p) & 6.82 & 6.10 & 5.89 \\
R/\Delta_{\text{min}}(c-c) & 9.35 & 9.64 & 10.03 \\
B_{\text{max}}/B_0 & 2.49 & 2.57 & 2.63 \\
(0.3x0.3) & 2.57 & 2.63 & 2.85 \\
\end{array}
\]

* Detailed discussion: 14th SOFE/PPPL-3886, ARIES-CS project meeting, September 2003, Atlanta, GA
$B_{\text{max}}$ increases as $A_c$ decreases, but large increases occur only for $A_c < 6$. $A_c = \frac{R}{\Delta_{\text{min}}(c-p)}$

R = 8.25 m, B = 6.5 T

Coil cross section

- 0.3x0.3
- 0.4x0.4
- 0.5x0.5
- 0.6x0.6
- 0.7x0.7
Comparison of two coil sets at R=8.25 m

\[ A_c = 6.8 \]
\[ \Delta_{\text{min}(c-p)} = 1.2 \text{ m} \]
\[ \Delta_{\text{min}(c-c)} = 0.88 \text{ m} \]
\[ I_{\text{max}} = 15.9 \text{ MA @6.5T} \]

\[ A_c = 5.9 \]
\[ \Delta_{\text{min}(c-p)} = 1.4 \text{ m} \]
\[ \Delta_{\text{min}(c-c)} = 0.83 \text{ m} \]
\[ I_{\text{max}} = 16.4 \text{ MA @6.5T} \]
Contours of distance from LCMS to the winding surface. R=8.25 m

$A_c = 6.8$

$A_c = 5.9$
N3ARE – An NCSX-like configuration with good QA, $\alpha$ confinement and MHD stability characteristics. A bias is introduced in the magnetic spectrum in favor of $B(0,1)$.

Plane and perspective views of the last LCMS geometry and $|B|$ in real space.
Comparison of magnetic spectrum of LI383 and N3ARE showing the distinctive feature in N3ARE – enhanced $B(0,1)$ and $B(1,1)$, reduced $B(2,1)$ and $B(3,2)$.

Eight major non-axisymmetric components in the magnetic spectrum plotted as function of normalized toroidal flux.

\[
B = \sum B_{mn} \cos (m\theta - |n - m|\varphi)
\]

\[
\text{noise} = \sqrt{\frac{\text{magnetic energy, nonsymmetric components}}{\text{magnetic energy, symmetric components}}}
\]
N3ARE has significantly lower effective helical ripples as calculated by NEO.
N3ARE has significantly better energetic particle confinement.

LI383 -- $\alpha$ energy loss fraction ~27%

N3ARE -- $\alpha$ energy loss fraction ~10%

Particle loss as function of time.
Cumulative particle loss.
Energy loss distribution.
Scatter plot of the escaping particles on the LCMS showing the structure of loss bands in \((\theta, \phi)\).
The external kinks and infinite-n ballooning modes are marginally stable at 4% $\beta$ in both configurations. The following shows the ballooning eigenvalue versus the normalized toroidal flux for the two cases.

LI383, $\zeta=60$, $\theta=0$

N3ARE, $\zeta=0$, $\theta=0$
The rotational transform of N3ARE is similar to that of NCSX so that the quality of equilibrium surface is expected to be similar.

Rotational transform as function of toroidal flux.
KQ26Q – modification of NCSX rotational transform to improve the robustness of flux surface integrity.

Plane and perspective views of the last LCMS geometry and $|B|$ in real space.
The external transform is increased to remove the $m=6$ rational surface and to move the $m=5$ surface to the core (relative to NCSX).
KQ26Q has good equilibrium flux surface quality, although the remnant of the m=4 islands may be a concern in free-boundary plasma reconstruction and in coil designs.

Equilibrium calculated by PIES @4% $\beta$.

Poincaré plot in r-$\theta$ at $\varphi=0$.

In Cartesian

Equilibrium calculated by VMEC
Minimizing non-axisymmetric residues and effective ripples resulted in good quasi-axisymmetry in KQ26Q. The effective ripple @s=1 is 0.7% at 4% $\beta$ and the $\alpha$ loss is ~7% in one slowing down time in the model calculation.

Eight major non-symmetric components in the magnetic spectrum plotted as function of normalized toroidal flux.

Max. “noise” content ~3.3% @s=1

$\varepsilon$-effective as function of normalized toroidal flux
KQ26Q may be unstable to free-boundary modes for $\beta > 4\%$ primarily due to current driven forces at the $m=3$, $n=2$ resonance, but it could be made stable with more flux surface shaping to improve the local shear. It may also be made more stable by choosing more optimized pressure and current profiles.

These modes may be stabilized by further shaping.
II. MHH2 with low aspect ratios

MHH2-K14 with aspect ratio ~2.65 having low field ripples and excellent confinement of $\alpha$ particles.

Plane and perspective views of the last LCMS geometry and $|B|$ in real space.

1) FS&T, 47, 3, 400 (2005), 2) SOFE 2005, 3) ARIES-CS project meeting February and June 2005.
MH2-K14 is a configuration of the ultra-low A family. It has a rising rotational transform profile in configuration optimization consistent with that expected with the bootstrap current and without any other driven currents.

LCMS in four toroidal angles over half period. Rotational transform as function of toroidal flux.

Expected at 5% $\beta$ with NCSX-like pressure/current profile

External transform due to plasma shaping

Assumed in configuration optimization
It is slightly unstable to both low- and high-n internal modes for $\beta \sim 4-5\%$.

**Low-n modes**  \[ \gamma \cdot R/v_A \sim 0.0009 \]

- $m=2, n=1$  
- $m=4, n=1$  
- $m=5, n=3$  
- $m=6, n=3$  
- $m=7, n=3$  

Radial displacement eigenfunction

**Infinite-n ballooning modes** (Cobra calculation)

\[ p \propto (1-s^{1.5})^{1.5} \]

- $5\% \beta$  
- $3\% \beta$  
- $2\% \beta$
MHH2-K14 may be also unstable to the external modes for $\beta>5\%$ according to the Terpsichore calculation, primarily due to modes of intermediate toroidal mode numbers 5 and 7.
MHH2-K14 has reasonably good flux surface integrity. While islands of the lowest orders, m=4, 5, 6, do not contribute to significant flux loss, the proximity of islands of intermediate mode numbers degrades the quality of the flux surfaces.

MHH2-K14 @ 5% $\beta$ with linear, monotonically increasing iota profile. Poincaré plot in r-$\theta$ coordinates at $\varphi=0$.

Poincaré plots in Cartesian coordinates at three different toroidal angles.
A modular coil design for MHH2-K14 (K14LA) via three stages of optimization.

No. of Coils: 8/period
Different Types of Coils: 4
$R/\Delta_{\text{min}}$ (coil-plasma)=5.5
$R/\Delta_{\text{min}}$ (coil-coil)=10.3
$I/R-B$ (max)=0.32 MA/m-T
$B(\text{max})/B(0) = 2.0$ for 0.4 m x 0.4 m conductor
Comparison of rotational transforms, reconstructed with K14LA versus the original fixed-boundary MHH2-K14, showing that both internal and external transforms are mostly recovered in the free-boundary equilibrium.

Rotational transform versus normalized toroidal flux. Left frame: free-boundary equilibrium due to K14LA. Right frame: fixed-boundary equilibrium.
Residues in the magnetic spectrum plotted as function of the normalized toroidal flux showing the excellent QA of the K14LA equilibrium.

- Free–boundary plasma, K14LA coils:
  - Effective ripple <0.8%
  - $\alpha$ energy loss in model calculation < 5%

Noise ~0.4%

Noise 2.2%
Magnetic field strengths plotted along several segments of field lines indicate that there are few secondary ripple wells for $r/a<0.7$. Secondary ripples are mostly on the high field side of the configuration.

$|B|$ versus poloidal angle $\theta$ in radians along field lines starting @ $\varphi=0$, $\theta=0$. 
III. SNS family of configurations

KJC167 – a showcase with essentially flat iota profile, demonstrating the existence of excellent flux surface integrity in QAS.

Plane and perspective views of last LCMS geometry and |B| in real space.

More detail in ARIES-CS project meeting June, 2004 and June 2005, Madison, WI
KJC167 is a 3 field-period, aspect ratio 6 configuration of the SNS family in which the iota profile is selected to minimize the impact of low order resonance on the flux surface integrity. In this case, the external iota has a strong negative shear, but the iota at operating $\beta$ is expected to have a small but positive shear in most of the plasma volume.
Excellent quality of flux surfaces is observed in most of the plasma for KJC167 at 6% $\beta$ as seen below based on a PIES calculation.

Equilibrium calculated by PIES @6% $\beta$.

Poincaré plot in r-θ at $\varphi=0$.

PIES and VMEC solutions are consistent.

Equilibrium calculated by VMEC.
Minimizing non-axisymmetric residues and effective ripples resulted in good quasi-axisymmetry. The effective ripple @s=1 is only 0.35% at 6% β and the overall “noise” is <2.5%. Loss of α energy is ~8% in one slowing down time in our model calculation.

\[ B = \sum B_{mn} \cos(m\theta - |n - m|\varphi) \]

\[
\text{noise} = \sqrt{\frac{\text{magnetic energy, nonsymmetric components}}{\text{magnetic energy, symmetric components}}} \]
KJC167 may be unstable to free-boundary modes for $\beta \sim 6\%$ according to the Terpsichore calculation primarily due to the $m=2$, $n=1$ mode, but it could be made stable with more flux surface shaping to improve the local shear. It may also be made more stable by choosing more optimized pressure and current profiles.
A proposed design for the modular coils is to have 6 coils/period with coil aspect ratio $R/\Delta_{\text{min}}(C-P) \sim 6$. The example given here, KJC167-M05, based on equal coil currents, has smooth contours with small toroidal excursion.

Coil contours viewed on “U-V” plane of the winding surface in one field period.
Comparison of KJC167-M05 and NCSX-M50 showing that the complexity of the two designs is of the same level.
Summary & Conclusions

• We have extended the NCSX-class of configurations for better alpha confinement and surface integrity.

• We have identified and developed new classes of configurations with smaller aspect ratios, better QA and more robust surface quality.

• We have shown that a reasonably large separation between plasma and coils \((R/\Delta_{\text{min}}<6)\) is achievable and coil ripples may be controlled with as small as 16 coils.

• We have demonstrated the richness of the QA magnetic topology, the flexibility in configuration optimization in improving the plasma engineering performance and, therefore, the potential of QA devices as candidates of compact power producing reactors.

• The most attractive configurations will ultimately be determined by results of systems optimization and other constraints arising from engineering designs.

• It is critical to obtain physics data base so as to allow a complete integration of our configuration optimization objectives.