Aries-CS
Physics Issues & Optimization Code

M.C. Zarnstorff
PPPL

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Outline

- Overview: stellarator design & opportunities
- Issues, assumptions, and design goals
- Tools
- Experience
- Summary & Next steps
Stellarator Advances

• Understanding of how to design for orbit confinement, good flux surfaces
• Numerical design to obtain desired physics properties
• Experience in accurately constructing experiments at a range of scales (CE -> PE), with good confinement and stability

Allows effective use of Stellarator Advantages:
  – Steady-state compatible, lack of need for external current drive
  – Disruptions typically not observed, can be avoided by design.
  – 3D Shaping, to obtain desired physics properties:
    high-beta stability, good confinement
Two strategies for Orbit Confinement in 3D

3D shape of standard stellarators ⇒
   field lines and particle orbits can have resonant perturbations,
       become stochastic ⇒ lost
B is bumpy every direction ⇒ rotation is strongly damped

• ‘quasi-symmetry’
   – Boozer (1983) Drift orbits & neoclassical transport depends on variation of
     |B| within flux surface, not the vector components of B!
   – If |B| is symmetric in “Boozer” coordinates, get confined orbits like tokamak
     ⇒ neoclassical transport very similar to tokamaks, undamped rotation

• Quasi-poloidal, Non-symmetric drift-orbit omnigeneity, linked mirrors…
   – Toroidal and helical drifts cancel; align drift orbit with flux surface
   – If could be done perfectly, would result in |B| independent of poloidal angle
   – Principle of W-7X, new German superconducting experiment (A=11)
   – Principle of QPS design
Optimized Stellarator Design Process

- Fixed Boundary Equilibrium Optimization
  - In depth analysis
    - Flux surface quality
    - Transport
    - Stability
  - Engineering Analysis
- Coil Design (to reproduced Fixed Bdry Equilibrium)
- Free Boundary Analysis
  - Robustness/Flexibility
  - Discharge Evolution
  - Transport, stability, flux surface quality
    - Engineering
    - Edge analysis

- Process as first developed for W7-X, used on HSX
  - has been extended to address finite $\beta$, current, and low A for NCSX & QPS
Helically Symmetric Experiment (HSX): Neoclassical Transport Reduction via Quasi-Helical Symmetry

In Boozer coordinates, magnetic field looks like straight helix

First test of quasi-symmetry started operation in 2000

R=1.2 m, B=1 T, 4 periods, R/⟨a⟩ = 8

Univ. of Wisconsin
New Design Process (NCSX, QPS)

- Fixed Boundary Equilibrium Optimization
- Find right neighborhoods
  - In depth analysis
    - Transport, stability, flux surface quality
    - Engineering Analysis
    - Edge analysis
- Coil Design
  - (to reproduce desired Equilibrium)
- Free Boundary Equilibrium Optimization of coils
- Healing of Islands
  - Free Boundary Analysis
    - Robustness/Flexibility
    - Discharge Evolution

- Only possible due to availability of parallel high-speed computers
NCSX Plasma Configuration Has Attractive Physics

• 3 periods, $R/\langle a \rangle = 4.4$, $\langle \kappa \rangle \sim 1.8$

• Quasi-axisymmetric: low helical ripple transport, low flow damping

• Passively stable at $\beta = 4.1\%$ to kink, ballooning, vertical, Mercier, neoclassical-tearing modes; without conducting walls or feedback systems.

• Steady state without current-drive

• 18 modular-coils (3 shapes)
  Full coil set includes PF coils & weak T

• Coils meet engineering criteria

Using Advances in Theory and Numerical modeling; parallel computing
(NERSC, ACL/LANL, Princeton/PPPL)
Choices, Choices

Stellarators provide very large configuration space

Need to identify
• Standard characteristics: size, B, A, adequate confinement, stability, coil-plasma separation for blankets & shield
• Number of field periods
• Orbit optimization strategy
• Rotational transform from coils
• Adequate alpha-particle confinement
  – What loss level is tolerable? Useful?
• Number & topology of coils, limit on bend radii &c.

Stellarators are different, offer different possible choices
e.g. is MHD marginal stability necessary?
Global Modes appear at intermediate $\beta$-Values
Pressure driven $(m,n) = (2,1)$ Modes around $\iota = 1/2$

X-Ray Tomograms reveal Ballooning Type Perturbation (always largest on outboard side)
New Design Process

- Fixed Boundary Equilibrium Optimization
  - Coil Optimization (to reproduced desired Equilibrium)
    - Free Boundary Equilibrium Optimization of coils
      - Healing of Islands
        - Free Boundary Analysis
          - Robustness/Flexibility
          - Discharge Evolution
            - Find right neighborhoods
              - In depth analysis
                - Transport, stability, flux surface quality
                - Engineering Analysis
                - Edge analysis

- Only possible due to availability of parallel high-speed computers
Primary Tool: Numerical Optimization
STELLOPT (ORNL & PPPL)

3D Equilibrium Calc. (VMEC)

MHD Stability high- & low-n

Orbit Confinement

Flux Surface Quality

Expected Coil Characteristics

Transport (simple)

... 

Adjust Plasma Shape
(modified Levenberg-Marquardt, Differential Evolution, Genetic)

Optimization Variables
Can choose to vary:
• Plasma Shape
  ~ 30-70 free parameters

• Coil Shapes
  ~ 200 – 400 f.parameters

• Coil Currents
  ~ 8 free parameters
Coil Design Process

\[ \mathbf{n} \]
Plasma Surface Normal \((\mathbf{B}_{\text{total}} \cdot \mathbf{n}=0)\)

\[ \mathbf{J}_{\text{Plasma}} \]
From VMEC

\[ \mathbf{B} \cdot \mathbf{n} \]
From Plasma \((\mathbf{B}_{\text{NORM}})\)

\[ \Sigma \mathbf{B} \cdot \mathbf{n} = 0 \]

\[ \mathbf{B} \cdot \mathbf{n} \]
From Coils

\[ \mathbf{J}_{\text{Coils}} \]
• Filaments on winding surface

\[ \mathbf{L} \]
Inductance Matrix Biot-Savart

**Vary “L” until** \( \mathbf{L} \cdot \mathbf{J}_{\text{Coils}} = \mathbf{B} \cdot \mathbf{n}(\text{Coils}) \approx -\mathbf{B} \cdot \mathbf{n}(\text{Plasma}) \)**
COILOPT – flexible Coil Optimization Tool
(D. Strickler, L. Berry, S. Hirshman, ORNL)

• Varies filament coil shapes within a winding surface
• Varies winding surface shape
• Can vary coil current. Can deal with different coil topologies (modular, saddle, PF)

For fixed number of coils, targets:
– \( B_{\text{normal}} \) mismatch
– Engineering criteria: bend radius, separation
– Coil-plasma separation
Established Equilibrium Codes Used

- VMEC - an ‘inverse’ equilibrium solver, which solves directly for the shape of the flux surfaces. Representation presumes that the flux surfaces are simply connected, without islands or stochastic regions. (Hirshman, ORNL)

- PIES - is a ‘forward’ equilibrium solver, directly calculating the 3D magnetic field and current distribution, including simulating the effect of islands and stochastic regions by flattening $\nabla p$. Flux surface topology and shape determined by integrating the field-line orbits. (Reiman, Monticello, et al, PPPL)

- Both well benchmarked against 2D equilibrium codes, each other, and against available other 3D equilibrium codes.
- VMEC compares well with SXR tomography on W7-AS.
- VMEC used for physics optimization,
  PIES used for island analysis and healing
Stability Codes and Their Validation

Equilibrium code

Stability code

MHD modes

Vertical mode (N=0)
External kink modes (N=0 & 1)

high-n ballooning modes (n >>1)

infinite-n ballooning modes

VMEC [1]

TERPSICHORE [2]
CAS3D [3]

COBRA [4]
VVBAL [5]

Cobra solves the ideal ballooning mode equation for eigenvalue \( \gamma^2 \):

\[
\rho \gamma^2 \frac{k_\perp^2}{B^2} \Phi - B \cdot \nabla \frac{k_\perp^2}{B^2} B \cdot \nabla \Phi - \frac{p'}{B^2} (k_\perp \times B) \cdot \kappa \Phi = 0 \quad (1)
\]

where \( k_\perp = \nabla \phi - q(\psi) \nabla \theta - q'(\theta - \theta_k) \nabla \psi \).

Terpsichore-VVBAL solves a modified ballooning mode equation for eigenvalue \( \lambda \):

\[
B \cdot \nabla \frac{k_\perp^2}{B^2} B \cdot \nabla \Phi + (1 - \lambda) \frac{p'}{B^2} (k_\perp \times B) \cdot \kappa \Phi = 0 \quad (2)
\]

where \( (1 - \lambda) \) is an artificial multiplier to the pressure-gradient term and \( \lambda > 0 \) for instability.

Note that the eigenvalues are defined differently in two codes but the marginal stability points are the same.

\( \gamma = \gamma(s, \theta_k, \alpha) \) with s being flux label, \( \theta_k \) being the radial wave number and \( \alpha \) being the field line variable.
The Terpsichore and CAS3D are 3D ideal MHD stability codes that determine stability by minimizing the plasma potential energy:

$$\delta W_p = \frac{1}{2} \int d^3 x [\delta B_\perp^2 + (\delta B_\parallel - B \frac{\xi \cdot \nabla p}{B^2})^2 + j_\parallel \cdot \xi \times \delta B - 2\xi \cdot \nabla p \cdot \xi \cdot \kappa]$$

- Both codes use a finite element method for radial discretization and Fourier decomposition in poloidal and toroidal angles.
- The Terpsichore treats vacuum as a pseudo-plasma. The CAS3D uses Green’s function method to solve the vacuum problem and thus can evaluate stability without a conducting wall.
- The Terpsichore code is used in the optimizer for sake of speed.
Thermal Confinement

• In Stellopt, have several targets available:
  * NEO calculations of effective-helical ripple ($\epsilon_{\text{eff}}$)
    – DKES diffusion coefficient for a single particle energy
    – RMS-sum of undesired B-harmonics
    – “Pseudo-symmetry” – secondary magnetic wells
    – $J$ contour alignment

• For detailed analysis, use:
  – Global scaling laws
    • ISS95 from a diverse set of stellarators
    • Equivalent-ITER97P, for quasi-axisymmetric stellarators
  – DKES calculations for full distribution function
  – Monte-Carlo simulations of neoclassical transport
  – Turbulence simulations almost available
DKES code (Hirshman) predictions confirmed by W7-AS (Maaßberg). Monoenergetic diffusivities are strongly reduced by $E_r$; and asymptotically approach the axisymmetric result. With the ambipolar $E_r$ the neoclassical ripple transport is negligible.
Fast Ion Orbit Confinement

Is not the same as thermal confinement, due to very low collisionality, large orbit displacements. Losses often due to stochastic orbits.

• Do not have fully adequate Stellopt target
  – Thermal confinement targets
  – J contour confinement & allignment
  – restricted Monte-Carlo simulation (v. expensive)

• For detailed analysis, use:
  – Monte-Carlo simulations of neoclassical transport
Flux-Surface Quality

• 3D Configurations typically have islands, stochastic regions

• So far, have not succeeded in developing useful target for Stellopt for targeting island

• Depend on removing resonant fields (and thus islands) as second optimization step
Island Removal Method

- Calculate coupling between plasma boundary shape and island widths by perturbation, using PIES
- Invert coupling matrix to find (small) shape modification to remove islands

(S. Hudson, D. Monticello, A. Reiman)
Coil Design Has Been Modified to Produce Good Surfaces

- “Dynamic healing” algorithm modifies coils in each PIES iteration to suppress targeted islands.
- Preserves engineering and physics constraints on coil curvature, minimum distance between coils, kink stability. (Stable up to n=45. Ballooning restabilized in startup scenario.)

Converged, free-boundary PIES calculation with healed coils.
Sum of effective island widths < 1%.

PIES calculation with original coils.
Continues to deteriorate as iteration proceeds.

plasma boundary in VMEC calculation with unhealed coils.
NCSX Design Experience

- Parameter space is very large. Many local minima.
- Optimization process is an exploration.
- We went through several stages, exploring and developing tools
  - Fixed boundary plasma
  - Coil designs
  - Optimized coil designs
  → For NCSX, each stage took more than a year. Generated many competing designs. Requires enormous amounts of computer time.
- It is important to identify goals, then explore for them directly
  - Best if the optimizer can directly target desired properties
Summary and Work to be Done

We have made enormous progress in developing a toolset for designing optimized stellarators. We can simultaneously target goals that were not approachable ~5 years ago.

To develop an optimized Stellarator reactor, we need to target some new goals

– Alpha-particle confinement
– COE – measure of reactor attractiveness
– Flux surface quality (if possible)

We should expect to spend some time exploring the landscape

– Need to make sure we have adequate computer resources