STELLARATOR REACTOR OPTIMIZATION AND ASSESSMENT

J. F. Lyon, ORNL

ARIES Meeting October 2-4, 2002
TOPICS

• Stellarator Reactor Optimization

• 0-D Spreadsheet Examples

• 1-D POPCON Examples

• 1-D Systems Optimization with Self-Consistent Electric Fields and Fueling

• Suggested Approach
Stellarators Have Complex 3-D Magnetic Fields and Coils

• No simple scaling laws for $\beta$ limits, confinement
  - depends on details of the magnetic configuration

• Divertor and maintenance requirements are more complex than for axisymmetric systems

• Systems codes must incorporate complex coil geometry and stellarator physics
  - complicated optimization and assessment
  - no geometry scalings possible
Reactor Core (Plasma and Coil) and Operating Point Optimizations are Separate

• Reactor core optimization leads to a \textit{fixed} plasma and coil geometry
  – integrated 3-D plasma/coil optimization code
  ⇒ plasma shape, aspect ratio, coil geometry
  ⇒ β limits, helical ripple, edge geometry
  ⇒ plasma-coil and coil-coil spacings, etc.

• Operating point optimization leads to plasma parameters, profiles, field and component sizes
  – 1-D systems code incorporating complex plasma and coil geometry and stellarator physics modules
Minimum Reactor Size Is Determined by $\Delta$

- A configuration is characterized by the ratios $A_\Delta = R_0/\Delta$, $A_p = R_0/<a>$, and $B_{\text{max}}/B_0$.

- The minimum reactor size is set by $R_0 = A_\Delta(D + ct/2)$ where $D$ is the space needed for scrapeoff, first wall, blanket, shield, coil case, and assembly gaps.

- Cost $\propto$ surface area $\propto A_\Delta^2/A_p$.  

Center of Coil Winding Surface

Major Radius $R_0$

Plasma Surface Ave. Radius $<a>$

Minimum Distance $\Delta$ between Plasma Edge and Center of Coil Winding Surface

$B_0$

$\Delta$

ct = coil thickness

$B_{\text{max}}$

Plasma
Lowest $<R>/<a>$ Does Not Necessarily Lead to the Most Compact Reactor

- For most reactor studies (ARIES, HSR, SPPS) $<a> = 1.7\text{-}1.8 \text{ m}$
  $\Rightarrow$ lowest $A_p = <R>/<a>$
  - for stellarator configuration with $A_p = 2.6$, this would give $<R> \approx 4.6 \text{ m}$, which is impossible

- The argument is OK for configurations where distance $\Delta$ between LCFS and coil center is *not* a hard constraint
  - OK for *truly* axisymmetric systems, *not* for QA or QP systems
  - for CS’s there is a maximum feasible $\Delta$ for a given $<R>$ before
    * the coils become too kinky and not buildable
    * $B_{\text{max}}/B_0$ becomes large

- A certain distance $D \approx 1.6 \text{ m} \leq \Delta$ is needed between LCFS and coil center
  $\Rightarrow$ minimum $<R> = A_\Delta D$ where $A_\Delta = <R>/\Delta$
0-D Spreadsheet Calculations

• **Fixed plasma and coil geometry**
  – $R/a$, $\nu(2/3 \ a)$, $R/\Delta$, $B_{\text{max}} / B_0$

• **Input parameters**
  – max H-ISS95, max $\beta$, max $T(0)$
  – max $B_{\text{max}}$, target $P_{\text{fusion}}$, max neutron wall loading ($\Gamma_n, \text{max}$)

• **Minimize $R$ for target $P_{\text{fusion}}$ by varying $n$ and H-ISS95 with constraints: parameters $\leq$ max. allowed values**
  – H-ISS95, $\beta$, $T(0)$, $n/n_{\text{Sudo}}$
  – plasma-coil distance, $j_{\text{coil}}$, $\Gamma_n$

• **Calculated quantities**
  – $R$, $a$, $n$, $T$, $\beta$, $v^*$, H-ISS95
  – plasma-coil distance, coil $j$, coil thickness, $P_{\text{fusion}}$, $\Gamma$

• **Useful for size scaling for fixed plasma and coil geometry and comparing reactor configurations**
Extrapolation of Compact Stellarators to a Reactor

• Vary distance $\Delta$ for compact stellarator configurations
  – calculate sheet-current solution at distance $\Delta$ from plasma that recreates desired plasma boundary
  – calculate $B_{\text{max}}/B_0$ at distance $ct/2$ radially in from current sheet

• Choose maximum credible distance $\Delta \Rightarrow R_0 = A_\Delta(D + ct/2)$

• $R_0^3 \propto P_{\text{fusion}}/B_0^4$, so want high $B_0$ for smaller reactor; however
  – $B_0$ decreases with increasing $\Delta$ ($B_{\text{max}}/B_0$ increases)
  – Coil complexity (kinks) increases with increasing $\Delta$

• Choose minimum $ct/2$ that satisfies two constraints
  – Ampere’s law: $B_0 = 2\mu_0 N j ct^2/(2\pi R_0)$; coil aspect ratio = 2 assumed
  – $B_0 = (16 \text{ T})/(B_{\text{max}}/B_0)$; $B_{\text{max}}/B_0$ increases as $ct$ decreases

• $B_{\text{max}}/B_0$ is larger for actual modular coils, so use $1.15B_{\text{max}}/B_0$

• Need to redo for real modular coils
### Scaled 1-GW Compact Stellarator Reactors

with $B_{\text{max}} = 12$ T, $\langle \beta \rangle \leq \beta_{\text{limit}}$, H-95 $\leq 5$

<table>
<thead>
<tr>
<th></th>
<th>QA#1</th>
<th>QA#2</th>
<th>QP#1</th>
<th>QP#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma aspect ratio $R/a_p$</td>
<td>2.96</td>
<td>4.4</td>
<td>2.70</td>
<td>3.70</td>
</tr>
<tr>
<td>Volume average $\beta$ limit $\langle \beta \rangle_{\text{limit}} (%)$</td>
<td>4</td>
<td>4.1</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td><strong>Average major radius $R$ (m)</strong></td>
<td>8.22</td>
<td>9.93</td>
<td>7.34</td>
<td>7.84</td>
</tr>
<tr>
<td>Average plasma radius $a_p$ (m)</td>
<td>2.78</td>
<td>2.26</td>
<td>2.72</td>
<td>2.12</td>
</tr>
<tr>
<td>Plasma volume $V_{\text{plasma}}$ (m$^3$)</td>
<td>1250</td>
<td>1000</td>
<td>1040</td>
<td>690</td>
</tr>
<tr>
<td>On-axis field $B_0$ (T)</td>
<td>5.41</td>
<td>5.65</td>
<td>5.23</td>
<td>5.03</td>
</tr>
<tr>
<td>$\tau_E/\tau_{E^{\text{ISS95}}}$ multiplier H-95</td>
<td>2.65</td>
<td>2.62</td>
<td>3.61</td>
<td>4.42</td>
</tr>
<tr>
<td><strong>Volume average beta $\langle \beta \rangle$ (%)</strong></td>
<td>4</td>
<td>4.1</td>
<td>4.6</td>
<td>6.2</td>
</tr>
<tr>
<td>Energy confinement time $\tau_E$ (s)</td>
<td>2.69</td>
<td>2.41</td>
<td>2.49</td>
<td>2.01</td>
</tr>
<tr>
<td>Vol.-ave. density $\langle n \rangle$ ($10^{20}$ m$^{-3}$)</td>
<td>1.31</td>
<td>1.50</td>
<td>1.40</td>
<td>1.70</td>
</tr>
<tr>
<td>Density-average $\langle T \rangle$ (keV)</td>
<td>11.1</td>
<td>10.8</td>
<td>11.3</td>
<td>11.5</td>
</tr>
<tr>
<td>Neutron wall load $\Gamma_n$ (MW m$^{-2}$)</td>
<td>1.34</td>
<td>1.37</td>
<td>1.54</td>
<td>1.85</td>
</tr>
</tbody>
</table>
Comparison with Other Stellarator Configurations

• The same assumptions were used with the plasma and coil configurations corresponding to the HSR, MHR-S, SPPS, QA and QP stellarator reactors.

• The modified “HSR*” had $R = 17.4$ m (instead of 22 m because $B_{\text{max}}$ was increased from 10.6 T to 12 T), $H_{95} = 3.06$, $\langle \beta \rangle = 4.9\%$, and $\Gamma_n = 1.24$ MW m$^{-2}$.

• The modified “MHR-S*” had $R = 18.6$ m (instead of 16.5 m because of the ARIES-AT blanket and shield assumptions), $H_{95} = 2.87$, $\langle \beta \rangle = 5\%$, and $\Gamma_n = 0.62$ MW m$^{-2}$.

• The modified “SPPS*” had $R = 20.8$ m (instead of 14.0 m because $B_{\text{max}}$ was decreased from 16 T to 12 T), $H_{95} = 3.13$, $\langle \beta \rangle = 5\%$, and $\Gamma_n = 0.60$ MW m$^{-2}$.

• For the same modeling assumptions, the compact stellarator configurations lead to reactors with a factor of 2 to 3 smaller major radius and a factor of 1.4 to 3 higher wall power loading.
1-D POPCON Calculations

- Fixed reactor parameters: $B_0$, $<R>$, $<a>$
- Plasma models for $\tau_E$ or $\chi$, radiation, etc.
- Fixed plasma assumptions: $\beta_{\text{limit}}$, $n_{\text{limit}}$, $\tau_{\text{He}}/\tau_E$, impurity fraction, $\alpha$ losses, etc.
- Calculates ignition contour and contours of $P_{\text{heating}}$ needed in $<n>$ vs. $<T>$ plane
- $<n>$ is the volume-averaged electron density
  $<T>$ is the density-averaged temperature
- Useful for understanding operating space, startup scenario, thermal stability
Typical QA Reactor POPCON Case

Operating Point
\(<n> = 9.5 \times 10^{19} \text{ m}^{-3}\)
\(<T> = 12.4 \text{ keV}\)
\(<\beta> = 3.6 \%\)
\(P_{\text{fus}} = 1.73 \text{ GW}\)

Saddle Point
\(<n> = 4.9 \times 10^{19} \text{ m}^{-3}\)
\(<T> = 6.1 \text{ keV}\)
\(<\beta> = 0.9 \%\)
\(P_{\text{aux}} = 12 \text{ MW}\)

Ignition minimum
\(<\beta> = 2.2 \%\)
\(P_{\text{fus}} = 0.6 \text{ GW}\)

- \(R = 9 \text{ m}, \ B_0 = 5 \text{ T} (B_{\text{coil}} = 12.7 \text{ T}), \ 2.5 \times \text{ISS-95}, \ 5\% \alpha\ \text{loss}\)
- \(\tau_{\text{He}}/\tau_E = 6 \Rightarrow 5.3\% \text{He}, \ n_{\text{DT}}/n_e = 0.83, \ Z_{\text{eff}} = 1.5\)
Operating Point Moves to Higher $<T>$ as ISS95 Multiplier $H$ Increases

- $R = 9 \text{ m}$, $B = 5 \text{ T}$, 5% $\alpha$ losses, $\tau_{\text{He}}/\tau_{\text{E}} = 6$
Operating Point Characteristics

- $R = 9 \text{ m}$, $B = 5 \text{ T}$, $\tau_{\text{He}}/\tau_{E} = 6$

Diagram:
- $P_{\text{fusion}}$ (GW thermal)
- Alpha-Particle Losses (%)

- Minimum $P_{\text{fus}}$
- Allowable $\alpha$ losses

- 5% $\alpha$ losses
Higher B Required at Lower H

- $R = 9 \text{ m}, 5\% \alpha \text{ losses, } \tau_{\text{He}}/\tau_{\text{E}} = 6$

$P_{\text{fus}} = 1.73 \text{ GW}_{\text{th}}$
Cost of Electricity Could Decrease with Plant Size

- Stellarator reactor example, similar for ARIES-RS
Systems Code Integrates Physics, Materials, Cost Models

- Detailed physics models for
  - alpha-particle heating and losses (Fokker-Planck with losses)
  - radiation (coronal line radiation, bremsstrahlung, cyclotron)

- Stellarator transport options (ISS95 + Shaing-Houlberg)
  - (a) 1-D evaluations with fixed profiles
  - (b) solve for $T_e(r)$ and $T_i(r)$ with fixed $n_e(r)$ and $E_r(r)$
  - (c) solve for $T_e(r)$, $T_i(r)$, $n_e(r)$ and $E_r(r)$ with fixed particle source

- ARIES magnet and reactor material assumptions
  - multi-region blanket and shield (except for divertor regions)
  - $B_{\text{max}}$ vs. $j$ in coil from ARIES studies
  - allowable stresses, reactor safety penalties, etc. from ARIES

- ARIES costing algorithms based on masses and cost per kg
  - ARIES-RS algorithms and accounts
Optimization Approach

- Minimize cost ($<R>$) or COE with constraints for a particular plasma and coil geometry using a nonlinear constrained optimizer with a large number of variables.

- Large number of constraints allowed, for example:
  - ignition margin, $\beta$ limit, H-ISS95, radial build, coil $j$ and $B_{\text{max}}$, plasma-coil distance, blanket and shielding thicknesses, TBR, access for divertors and maintenance, etc.

- Large number of configuration, plasma parameters, transport model, costing, and engineering model parameters.
1-D Transport in Systems Code

- Steady-state 1–D integral-differential equations for the heat and particle fluxes for the ions (D,T) and electrons are solved for $n_e(r)$, $n_i(r)$, $T_e(r)$, $T_i(r)$, and $E_r(r)$:

$$\rho q_j(\rho) = a_p \int p_j(\rho^*) \rho^* d\rho^* , \quad \rho \Gamma_j(\rho) = a_p \int s_j(\rho^*) \rho^* d\rho^* ; \quad j = \text{ions, electrons}$$

- Heat flux $q_j = -n_j \chi_j^T \nabla T_j - T_j \chi_j^n \nabla n_j - Z_j n_j \chi_j^\phi \nabla \phi$

- Particle flux $\Gamma_j = -n_j D_j^T \nabla T_j - T_j D_j^n \nabla n_j - Z_j n_j D_j^\phi \nabla \phi$

- The electric field is determined from the ambipolarity condition. The electric field $E$ enters both through an $E/B$ drift term in the denominators of $\chi$ and $D$, and directly through the sign-dependent $\nabla \phi$ term.

- The volumetric heat sources (and sinks) are the usual alpha-particle heating, electron-ion heat transfer, and radiation terms.

- The form for the particle source rate ($s$) is chosen to represent shallow or deep fueling of the plasma.
Cost of Electricity Depends on $A_\Delta^2/A_p$

- Minimized COE for fixed fusion power
Beta and Confinement Multiplier are Coupled

- Minimized COE for fixed fusion power
Settles on Constant Value below $\beta$ limit

- Minimized cost of electricity
1-D Systems Optimization Calculations

- Reference parameters: \( R_0 = 12 \text{ m}, \ a_p = 1.5 \text{ m}, \ B_0 = 7 \text{ T}, \ P_{\text{fus}} = 3 \text{ GW (thermal)}, \) edge helical field ripple \( \epsilon_h(r = a_p) = 0.1. \)
Sensitivity to Parameter Assumptions

<table>
<thead>
<tr>
<th>$\varepsilon_h(1)$</th>
<th>$\chi_{\text{anom}}$ (m$^2$/s)</th>
<th>$\langle n \rangle$ $(10^{20}$ m$^{-3}$)</th>
<th>$n_D(0)$ $(10^{20}$ m$^{-3}$)</th>
<th>$T_e(0)$ (keV)</th>
<th>$T_i(0)$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.60</td>
<td>1.85</td>
<td>3.53</td>
<td>10.07</td>
<td>9.43</td>
</tr>
<tr>
<td>0.2</td>
<td>1.49</td>
<td>2.11</td>
<td>3.92</td>
<td>8.97</td>
<td>8.51</td>
</tr>
<tr>
<td>0.3</td>
<td>0.81</td>
<td>2.11</td>
<td>4.99</td>
<td>8.08</td>
<td>7.72</td>
</tr>
<tr>
<td>0.4</td>
<td>0.66</td>
<td>2.21</td>
<td>6.08</td>
<td>7.31</td>
<td>7.09</td>
</tr>
<tr>
<td>0.6</td>
<td>0.43</td>
<td>2.01</td>
<td>7.93</td>
<td>7.00</td>
<td>6.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pf (GW)</th>
<th>$B_0$ (T)</th>
<th>$\alpha$ –loss</th>
<th>$\chi_{\text{anom}}$ (m$^2$/s)</th>
<th>$\langle n \rangle$ $(10^{20}$ m$^{-3}$)</th>
<th>$n_D(0)$ $(10^{20}$ m$^{-3}$)</th>
<th>$T_e(0)$ (keV)</th>
<th>$T_i(0)$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>no</td>
<td>2.60</td>
<td>1.85</td>
<td>3.53</td>
<td>10.07</td>
<td>9.43</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>yes</td>
<td>0.79</td>
<td>1.68</td>
<td>3.11</td>
<td>10.34</td>
<td>9.72</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>no</td>
<td>1.47</td>
<td>1.85</td>
<td>3.47</td>
<td>9.47</td>
<td>8.85</td>
</tr>
<tr>
<td>4.5</td>
<td>7</td>
<td>no</td>
<td>4.32</td>
<td>2.27</td>
<td>4.32</td>
<td>10.22</td>
<td>9.57</td>
</tr>
<tr>
<td>$\chi_{\text{anom}} \propto 1 + 19\rho^3$</td>
<td>0.23</td>
<td>1.25</td>
<td>3.21</td>
<td>13.92</td>
<td>12.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here $\chi_{\text{anom}} (\propto 1/n)$ is the largest value for ignition
1-D Systems Optimization Calculations

- Electron Density ($10^{20} \text{ m}^{-3}$)
- Electron Temperature (keV)
- Electric Field (kV/m)

- $n(1)/n(0) = 0.2$
- $\varepsilon_h(1) = 0.1$
- $\varepsilon_h(1) = 0.4$, $0.3$, $0.2$
Lessons Learned

• 3-D stellarator magnetic fields means more complex divertor and maintenance geometry, no simple scaling laws, no geometry scaling studies with a simple systems code

• Systems codes must incorporate complex coil geometry and stellarator physics
  – optimization and assessment more complicated

• Geometry scaling studies are not possible
  – plasma: shape, aspect ratio, plasma profiles
  – coils: plasma-coil and coil-coil spacings
Most Important Measure of Reactor Attractiveness is COE

<table>
<thead>
<tr>
<th>Reactor</th>
<th>Type</th>
<th>( R_0 / &lt;a&gt; )</th>
<th>( R_0 ) (m)</th>
<th>( p_{wall} ) (MW/m²)</th>
<th>COE (mills/kWh)</th>
<th>( Q_{eng} )</th>
<th>( B_{max}/B_0 )</th>
<th>( B_0 ) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W7-X based HSR</td>
<td>high-A stellarator</td>
<td>12.2</td>
<td>22</td>
<td>0.5</td>
<td>&gt;110</td>
<td>2.11</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>W7-X like SPPS</td>
<td>modular stellarator</td>
<td>8.6</td>
<td>13.9</td>
<td>1.3</td>
<td>75</td>
<td>19.3</td>
<td>2.94</td>
<td>4.9</td>
</tr>
<tr>
<td>ARIES-IV</td>
<td>2nd stability tokamak</td>
<td>2.8</td>
<td>6.0</td>
<td>2.7</td>
<td>68</td>
<td>5.2</td>
<td>2.09</td>
<td>7.6</td>
</tr>
<tr>
<td>ARIES-RS</td>
<td>reverse shear tokamak</td>
<td>3.1</td>
<td>5.5</td>
<td>4.0</td>
<td>76</td>
<td>5.9</td>
<td>1.98</td>
<td>8.0</td>
</tr>
<tr>
<td>ARIES-ST</td>
<td>spherical tokamak</td>
<td>0.87</td>
<td>3.2</td>
<td>4.1</td>
<td>&gt;76</td>
<td>3.1</td>
<td>3.55</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Higher value of \( Q_{eng} \) can compensate for \( R_0, p_{wall} \)
## Reactor Comparisons

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \frac{R_0}{\Delta} )</th>
<th>( R_0 ) (m)</th>
<th>( B_{\text{max}}/B_0 )</th>
<th>( P_{\text{elect}} )</th>
<th>( p_{\text{wall}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QA C82</td>
<td>5.8</td>
<td>8.9</td>
<td>2.54</td>
<td>2.0</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>2.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QA A4.1</td>
<td>5.8</td>
<td>9.0</td>
<td>2.20</td>
<td>2.6</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>4.1</td>
<td>2.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QA C93</td>
<td>5.8</td>
<td>9.0</td>
<td>2.14</td>
<td>4.2</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>2.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARIES-RS tokamak</td>
<td>3.4</td>
<td>5.5</td>
<td>1.98</td>
<td>1.0</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPPS stellarator</td>
<td>7.0</td>
<td>13.9</td>
<td>2.94</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>8.6</td>
<td>1.6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Closer to ARIES-RS than SPPS
- \( B_{\text{max}} = 16 \) T and \( \langle \beta \rangle = 5\% \) leads to large \( P_{\text{elect}} \)
The Coils are the Key to a CS reactor

- Plasma-coil spacing $\Delta$ (for blanket and shielding) and coil bend radii $\rho$ are more important than the plasma configuration or aspect ratio
  - $R \propto \Delta$ (for blanket and shielding) and cost $\propto \Delta^2$
  - $\rho \Rightarrow B_{\text{max}}$ on the coils $\Rightarrow P_{\text{fusion}} \propto B_{\text{max}}^4$

- Can’t just enlarge an experiment to reactor size
  - Optimization is based on different needs
  - W 7-X, LHD, NCSX, QPS don’t extrapolate to good reactors
A Phased Approach

- Development of optimization tools and modules highest priority

- Need combined optimization of plasma and coil configurations
  - no point in optimizing plasma and then finding coils
  - need to include reactor physics (α losses, divertor, etc.)
  - minimum cost implies minimum major radius and simplest coils, *not* minimum plasma aspect ratio
  - the key parameters are minimum values of $R/\Delta$ and $B_{\text{max}}/B_0$; small $R/a$ is a secondary factor

- Concerns
  - pace of reactor concept development restricted by very limited funding ($200k$ PPPL, $70k$ ORNL)
  - have to proceed at slow pace or drop some parts